A P P E N D I X A
Review of Fundamental Concepts of Algebra

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Appendix A.1 Review of Real Numbers and Their Properties

■ You should know the following sets.

(a) The set of real numbers includes the rational numbers and the irrational numbers.

(b) The set of rational numbers includes all real numbers that can be written as the ratio p/q of two integers, where $q \neq 0$.

(c) The set of irrational numbers includes all real numbers which are not rational.

(d) The set of integers: $\{. \ . \ ., -3, -2, -1, 0, 1, 2, 3, . \ . \ .\}$

(e) The set of whole numbers: $\{0, 1, 2, 3, 4, . \ . \ .\}$

(f) The set of natural numbers: $\{1, 2, 3, 4, . \ . \ .\}$

■ The real number line is used to represent the real numbers.

■ Know the inequality symbols.

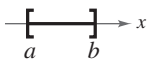
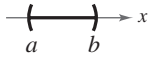
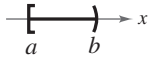
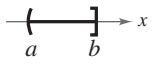
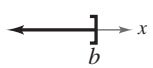
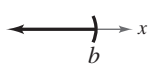
(a) $a < b$ means a is less than b.

(b) $a \leq b$ means a is less than or equal to b.

(c) $a > b$ means a is greater than b.

(d) $a \geq b$ means a is greater than or equal to b.

■

Interval Notation	*Inequality Notation*	*Graph*	*Type*
$[a, b]$	$a \leq x \leq b$		Bounded and Closed
(a, b)	$a < x < b$		Bounded and Open
$[a, b)$	$a \leq x < b$		Bounded
$(a, b]$	$a < x \leq b$		Bounded
$[a, \infty)$	$x \geq a$		Unbounded
(a, ∞)	$x > a$		Unbounded
$(-\infty, b]$	$x \leq b$		Unbounded
$(-\infty, b)$	$x < b$		Unbounded
$(-\infty, \infty)$	$-\infty < x < \infty$		Unbounded

■ You should know that $|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$.

■ Know the properties of absolute value.

(a) $|a| \geq 0$ (b) $|-a| = |a|$ (c) $|ab| = |a|\,|b|$ (d) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \neq 0$

—CONTINUED—

- The distance between a and b on the real line is $d(a, b) = |b - a| = |a - b|$.
- You should be able to identify the terms in an algebraic expression.
- You should know and be able to use the basic rules of algebra.
- Commutative Property
  - (a) Addition: $a + b = b + a$
  - (b) Multiplication: $a \cdot b = b \cdot a$
- Associative Property
  - (a) Addition: $(a + b) + c = a + (b + c)$
  - (b) Multiplication: $(ab)c = a(bc)$
- Identity Property
  - (a) Addition: 0 is the identity; $a + 0 = 0 + a = a$.
  - (b) Multiplication: 1 is the identity; $a \cdot 1 = 1 \cdot a = a$.
- Inverse Property
  - (a) Addition: $-a$ is the additive inverse of a; $a + (-a) = -a + a = 0$.
  - (b) Multiplication: $1/a$ is the multiplicative inverse of a, $a \neq 0$; $a(1/a) = (1/a)a = 1$.
- Distributive Property
  - (a) $a(b + c) = ab + ac$
  - (b) $(a + b)c = ac + bc$
- Properties of Negation
  - (a) $(-1)a = -a$
  - (b) $-(-a) = a$
  - (c) $(-a)b = a(-b) = -ab$
  - (d) $(-a)(-b) = ab$
  - (e) $-(a + b) = (-a) + (-b) = -a - b$
- Properties of Equality
  - (a) If $a = b$, then $a \pm c = b \pm c$.
  - (b) If $a = b$, then $ac = bc$.
  - (c) If $a \pm c = b \pm c$, then $a = b$.
  - (d) If $ac = bc$ and $c \neq 0$, then $a = b$.
- Properties of Zero
  - (a) $a \pm 0 = a$
  - (b) $a \cdot 0 = 0$
  - (c) $0 \div a = 0/a = 0$, $a \neq 0$
  - (d) $a/0$ is undefined.
  - (e) If $ab = 0$, then $a = 0$ or $b = 0$.
- Properties of Fractions ($b \neq 0, d \neq 0$)
  - (a) Equivalent Fractions: $a/b = c/d$ if and only if $ad = bc$.
  - (b) Rule of Signs: $-a/b = a/-b = -(a/b)$ and $-a/-b = a/b$
  - (c) Equivalent Fractions: $a/b = ac/bc$, $c \neq 0$
  - (d) Addition and Subtraction
    - 1. Like Denominators: $(a/b) \pm (c/b) = (a \pm c)/b$
    - 2. Unlike Denominators: $(a/b) \pm (c/d) = (ad \pm bc)/bd$
  - (e) Multiplication: $(a/b) \cdot (c/d) = (ac)/(bd)$
  - (f) Division: $(a/b) \div (c/d) = (a/b) \cdot (d/c) = (ad)/(bc)$ if $c \neq 0$.

Vocabulary Check

1. rational
2. irrational
3. absolute value
4. composite
5. prime
6. variables; constants
7. terms
8. coefficient
9. zero-factor property

1. $-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{2}, 0, 1, -4, 2, -11$

(a) Natural numbers: 5, 1, 2

(b) Whole numbers: 0, 5, 1, 2

(c) Integers: $-9, 5, 0, 1, -4, 2, -11$

(d) Rational numbers: $-9, -\frac{7}{2}, 5, \frac{2}{3}, 0, 1, -4, 2, -11$

(e) Irrational numbers: $\sqrt{2}$

2. $\sqrt{5}, -7, -\frac{7}{3}, 0, 3.12, \frac{5}{4}, -3, 12, 5$

(a) Natural numbers: 12, 5

(b) Whole numbers: 0, 12, 5

(c) Integers: $-7, 0, -3, 12, 5$

(d) Rational numbers: $-7, -\frac{7}{3}, 0, 3.12, \frac{5}{4}, -3, 12, 5$

(e) Irrational numbers: $\sqrt{5}$

3. $2.01, 0.666\ldots, -13, 0.010110111\ldots, 1, -6$

(a) Natural numbers: 1

(b) Whole numbers: 1

(c) Integers: $-13, 1, -6$

(d) Rational numbers: $2.01, 0.666\ldots, -13, 1, -6$

(e) Irrational numbers: $0.010110111\ldots$

4. $2.3030030003\ldots, 0.7575, -4.63, \sqrt{10}, -75, 4$

(a) Natural numbers: 4

(b) Whole numbers: 4

(c) Integers: $-75, 4$

(d) Rational numbers: $0.7575, -4.63, -75, 4$

(e) Irrational numbers: $2.3030030003\ldots, \sqrt{10}$

5. $-\pi, -\frac{1}{3}, \frac{6}{3}, \frac{1}{2}\sqrt{2}, -7.5, -1, 8, -22$

(a) Natural numbers: $\frac{6}{3}$ (since it equals 2), 8

(b) Whole numbers: $\frac{6}{3}$, 8

(c) Integers: $\frac{6}{3}, -1, 8, -22$

(d) Rational numbers: $-\frac{1}{3}, \frac{6}{3}, -7.5, -1, 8, -22$

(e) Irrational numbers: $-\pi, \frac{1}{2}\sqrt{2}$

6. $25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, \frac{1}{2}\pi, 7, -11.1, 13$

(a) Natural numbers: $25, \sqrt{9}, 7, 13$

(b) Whole numbers: $25, \sqrt{9}, 7, 13$

(c) Integers: $25, -17, \sqrt{9}, 7, 13$

(d) Rational numbers:

$25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, 7, -11.1, 13$

(e) Irrational numbers: $\frac{1}{2}\pi$

7. $\frac{5}{8} = 0.625$

8. $\frac{1}{3} = 0.\overline{3}$

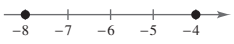
9. $\frac{41}{333} = 0.\overline{123}$

10. $\frac{6}{11} = 0.\overline{54}$

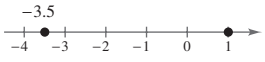
11. $-1 < 2.5$

12. $-6 < -2.5$

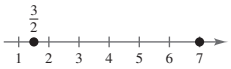
13. $-4 > -8$



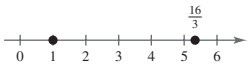
14. $-3.5 < 1$



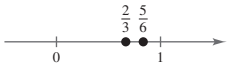
15. $\frac{3}{2} < 7$



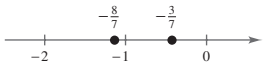
16. $1 < \frac{16}{3}$



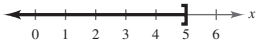
17. $\frac{5}{6} > \frac{2}{3}$



18. $-\frac{8}{7} < -\frac{3}{7}$

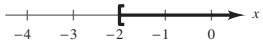


19. (a) The inequality $x \leq 5$ denotes the set of all real numbers less than or equal to 5.

(b) 

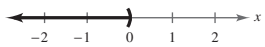
(c) The interval is unbounded.

20. (a) The inequality $x \geq -2$ denotes the set of all real numbers greater than or equal to -2.

(b) 

(c) The interval is unbounded.

21. (a) The inequality $x < 0$ denotes the set of all negative real numbers.

(b) 

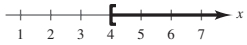
(c) The interval is unbounded.

22. (a) The inequality $x > 3$ denotes the set of all real numbers greater than 3.

(b) 

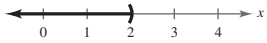
(c) The interval is unbounded.

23. (a) The interval $[4, \infty)$ denotes the set of all real numbers greater than or equal to 4.

(b) 

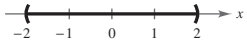
(c) The interval is unbounded.

24. (a) $(-\infty, 2)$ denotes the set of all real numbers less than 2.

(b) 

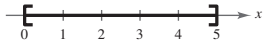
(c) The interval is unbounded.

25. (a) The inequality $-2 < x < 2$ denotes the set of all real numbers greater than -2 and less than 2.

(b) 

(c) The interval is bounded.

26. (a) The inequality $0 \le x \le 5$ denotes the set of all real numbers greater than or equal to zero and less than or equal to 5.

(b) 

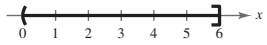
(c) The interval is bounded.

27. (a) The inequality $-1 \le x < 0$ denotes the set of all negative real numbers greater than or equal to -1.

(b) 

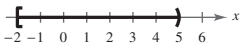
(c) The interval is bounded.

28. (a) The inequality $0 < x \le 6$ denotes the set of all real numbers greater than zero and less than or equal to 6.

(b) 

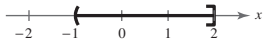
(c) The interval is bounded.

29. (a) The interval $[-2, 5)$ denotes the set of all real numbers greater than or equal to -2 and less than 5.

(b) 

(c) The interval is bounded.

30. (a) The interval $(-1, 2]$ denotes the set of all real numbers greater than -1 and less than or equal to 2.

(b) 

(c) The interval is bounded.

31. $-2 < x \le 4$

32. $-6 \le y < 0$

33. $y \ge 0$

34. $y \le 25$

35. $10 \le t \le 22$

36. $-3 \le k < 5$

37. $W > 65$

38. $2.5\% \le r \le 5\%$

39. $|-10| = -(-10) = 10$

40. $|0| = 0$

41. $|3 - 8| = |-5| = -(-5) = 5$

42. $|4 - 1| = |3| = 3$

43. $|-1| - |-2| = 1 - 2 = -1$

44. $-3 - |-3| = -3 - (3) = -6$

45. $\dfrac{-5}{|-5|} = \dfrac{-5}{-(-5)} = \dfrac{-5}{5} = -1$

46. $-3|-3| = -3(3) = -9$

47. If $x < -2$, then $x + 2$ is negative.

Thus $\dfrac{|x + 2|}{x + 2} = \dfrac{-(x + 2)}{x + 2} = -1.$

48. If $x > 1$, then $x - 1$ is positive.

Thus, $\dfrac{|x-1|}{x-1} = \dfrac{x-1}{x-1} = 1$.

49. $|-3| > -|-3|$ since $3 > -3$.

50. $|-4| = |4|$ since $|-4| = 4$ and $|4| = 4$.

51. $-5 = -|5|$ since $-5 = -5$.

52. $-|-6| < |-6|$ since $|-6| = 6$ and $-|-6| = -(6) = -6$.

53. $-|-2| = -|2|$ since $-2 = -2$.

54. $-(-2) > -2$ since $-(-2) = 2$.

55. $d(126, 75) = |75 - 126| = 51$

56. $d(-126, -75) = |75 - (-126)| = 51$

57. $d\left(-\frac{5}{2}, 0\right) = \left|0 - \left(-\frac{5}{2}\right)\right| = \frac{5}{2}$

58. $d\left(\frac{1}{4}, \frac{11}{4}\right) = \left|\frac{11}{4} - \frac{1}{4}\right| = \frac{5}{2}$

59. $d\left(\frac{16}{5}, \frac{112}{75}\right) = \left|\frac{112}{75} - \frac{16}{5}\right| = \frac{128}{75}$

60. $d(9.34, -5.65) = |-5.65 - 9.34| = 14.99$

61.

Budgeted Expense, b	*Actual Expense*, a	$\lvert a - b \rvert$	$0.05b$
\$112,700	\$113,356	\$656	$0.05(112{,}700) = \$5635$

Since \$656 < \$5635 but \$656 > \$500, the actual expense does not pass the "budget variance test."

62.

Budgeted Expense, b	*Actual Expense, a*	$\lvert a - b \rvert$	$0.05b$
\$9400	\$9772	\$372	$0.05(9400) = \$470$

Since \$372 < \$470 and \$372 < \$500, the actual expense does pass the "budget variance test."

63.

Budgeted Expense, b	*Actual Expense*, a	$\lvert a - b \rvert$	$0.05b$
\$37,640	\$37,335	\$305	$0.05(37{,}640) = \$1882$

Since \$305 < \$500 and \$305 < \$1882, the actual expense passes the "budget variance test."

64.

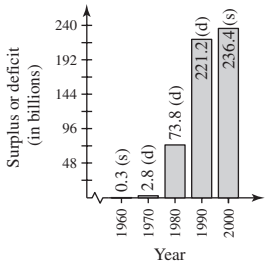
Budgeted Expense, b	*Actual Expense, a*	$\lvert a - b \rvert$	$0.05b$
\$2575	\$2613	\$38	$0.05(2575) = \$128.75$

Since \$38 < \$500, and \$38 < \$128.75, the actual expense passes the "budget variance test."

65. (a)

Year	Expenditures (in billions)	Surplus or Deficit (in billions)
1960	\$92.2	$\lvert 92.5 - 92.2 \rvert$ = \$0.3 surplus
1970	\$195.6	$\lvert 192.8 - 195.6 \rvert$ = \$2.8 deficit
1980	\$590.9	$\lvert 517.1 - 590.9 \rvert$ = \$73.8 deficit
1990	\$1253.2	$\lvert 1032.0 - 1253.2 \rvert$ = \$221.2 deficit
2000	\$1788.8	$\lvert 2025.2 - 1788.8 \rvert$ = \$236.4 surplus

(b)



66. Total: $2213 + 3290 + 4666 + 5665 + 9784 = 25{,}618$

Under 35: $\dfrac{2213}{25{,}618} \approx 0.0863857 = 8.63857\%$

8.63857% of $360^\circ \approx 31.1^\circ$

35–44: $\dfrac{3290}{25{,}618} \approx 0.1284253 = 12.84253\%$

12.84253% of $360^\circ \approx 46.2^\circ$

45–54: $\dfrac{4666}{25{,}618} \approx 0.18213756 = 18.213756\%$

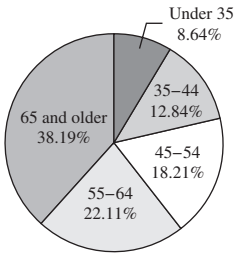
18.213756% of $360^\circ \approx 65.6^\circ$

55–64: $\dfrac{5665}{25{,}618} \approx 0.221133578 = 22.1133578\%$

22.1133578% of $360^\circ \approx 79.6^\circ$

65 and older: $\dfrac{9784}{25{,}618} \approx 0.38191896 = 38.191896\%$

38.191896% of $360^\circ \approx 137.5\%$



67. $d(x, 5) = |x - 5|$ and $d(x, 5) \le 3$, thus $|x - 5| \le 3$.

68. $d(x, -10) = |x + 10|$, and $d(x, -10) \ge 6$, thus, $|x + 10| \ge 6$.

69. $d(y, 0) = |y - 0| = |y|$ and $d(y, 0) \ge 6$, thus $|y| \ge 6$.

70. $d(y, a) = |y - a|$ and $d(y, a) \le 2$, thus $|y - a| \le 2$.

71. $d(326, 351) = |351 - 326| = 25$ miles

72. $d(48^\circ, 82^\circ) = |82^\circ - 48^\circ| = 34^\circ$

73. $7x + 4$

Terms: $7x, 4$

Coefficient: 7

74. $6x^3 - 5x$

Terms: $6x^3, -5x$

Coefficients: $6, -5$

75. $\sqrt{3}x^2 - 8x - 11$

Terms: $\sqrt{3}x^2, -8x, -11$

Coefficients: $\sqrt{3}, -8$

76. $3\sqrt{3}x^2 + 1$

Terms: $3\sqrt{3}x^2, 1$

Coefficients: $3\sqrt{3}$

77. $4x^3 + \dfrac{x}{2} - 5$

Terms: $4x^3, \dfrac{x}{2}, -5$

Coefficients: $4, \dfrac{1}{2}$

78. $3x^4 - \dfrac{x^2}{4}$

Terms: $3x^4, -\dfrac{x^2}{4}$

Coefficients: $3, -\dfrac{1}{4}$

79. $4x - 6$

(a) $4(-1) - 6 = -4 - 6 = -10$

(b) $4(0) - 6 = 0 - 6 = -6$

80. $9 - 7x$

(a) $9 - 7(-3) = 9 + 21 = 30$

(b) $9 - 7(3) = 9 - 21 = -12$

81. $x^2 - 3x + 4$

(a) $(-2)^2 - 3(-2) + 4 = 4 + 6 + 4 = 14$

(b) $(2)^2 - 3(2) + 4 = 4 - 6 + 4 = 2$

82. $-x^2 + 5x - 4$

(a) $-(-1)^2 + 5(-1) - 4 = -1 - 5 - 4 = -10$

(b) $-(1)^2 + 5(1) - 4 = -1 + 5 - 4 = 0$

83. $\dfrac{x+1}{x-1}$

(a) $\dfrac{1+1}{1-1}=\dfrac{2}{0}$

Division by zero is undefined

(b) $\dfrac{-1+1}{-1-1}=\dfrac{0}{-2}=0$

84. $\dfrac{x}{x+2}$

(a) $\dfrac{2}{2+2}=\dfrac{2}{4}=\dfrac{1}{2}$

(b) $\dfrac{-2}{-2+2}=\dfrac{2}{0}$

Division by 0 is undefined.

85. $x+9=9+x$

Commutative Property of Addition

86. $2\left(\dfrac{1}{2}\right)=1$

Multiplicative Inverse Property

87. $\dfrac{1}{(h+6)}(h+6)=1, h\neq -6$

Multiplicative Inverse Property

88. $(x+3)-(x+3)=0$

Additive Inverse Property

89. $2(x+3)=2x+6$

Distributive Property

90. $(z-2)+0=z-2$

Additive Identity Property

91. $1\cdot(1+x)=1+x$

Multiplicative Identity Property

92. $(z+5)x=z\cdot x+5\cdot x$

Right Distributive Property

93. $x+(y+10)=(x+y)+10$

Associative Property of Addition

94. $x(3y)=(x\cdot 3)y$ Associative Property of Multiplication

$=(3x)y$ Commutative Property of Multiplication

95. $3(t-4)=3\cdot t-3\cdot 4$

Distributive Property

96. $\frac{1}{7}(7\cdot 12)=\left(\frac{1}{7}\cdot 7\right)12$ Associative Property of Multiplication

$=1\cdot 12$ Multiplicative Inverse Property

$=12$ Multiplicative Identity Property

97. $\dfrac{3}{16}+\dfrac{5}{16}=\dfrac{8}{16}=\dfrac{1}{2}$

98. $\dfrac{6}{7}-\dfrac{4}{7}=\dfrac{6-4}{7}=\dfrac{2}{7}$

99. $\frac{5}{8}-\frac{5}{12}+\frac{1}{6}=\frac{15}{24}-\frac{10}{24}+\frac{4}{24}=\frac{9}{24}=\frac{3}{8}$

100. $\frac{10}{11}+\frac{6}{33}-\frac{13}{66}=\frac{60}{66}+\frac{12}{66}-\frac{13}{66}=\frac{59}{66}$

101. $12\div\frac{1}{4}=12\cdot\frac{4}{1}=12\cdot 4=48$

102. $-\left(6\cdot\frac{4}{8}\right)=-6\cdot\frac{1}{2}=-3$

103. $\dfrac{2x}{3}-\dfrac{x}{4}=\dfrac{8x}{12}-\dfrac{3x}{12}=\dfrac{5x}{12}$

104. $\dfrac{5x}{6}\cdot\dfrac{2}{9}=\dfrac{5x}{3}\cdot\dfrac{1}{9}=\dfrac{5x}{27}$

105. (a)

n	1	0.5	0.01	0.0001	0.000001
$5/n$	5	10	500	50,000	5,000,000

(b) The value of $5/n$ approaches infinity as n approaches 0.

106. (a)

n	1	10	100	10,000	100,000
$5/n$	5	0.5	0.05	0.0005	0.00005

(b) The value of $5/n$ approaches 0 as n increases without bound.

107. False. If $a<b$, then $\dfrac{1}{a}>\dfrac{1}{b}$, where $a\neq b\neq 0$.

108. False. The denominators cannot be added when adding fractions.

109. (a) $|u + v| \neq |u| + |v|$ if u is positive and v is negative or vice versa.

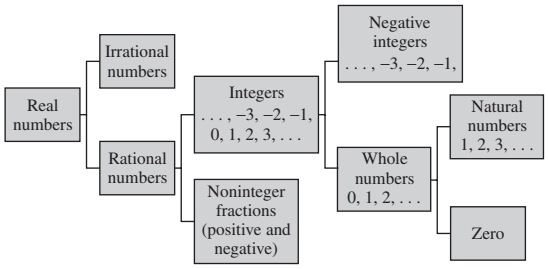
(b) $|u + v| \leq |u| + |v|$

They are equal when u and v have the same sign. If they differ in sign, $|u + v|$ is less than $|u| + |v|$.

110. Yes. y is nonnegative if $y \geq 0$. y is positive if $y > 0$.

111. The only even prime number is 2, because its factors are itself and 1.

112.



113. (a) Since $A > 0$, $-A < 0$. The expression is negative.

(b) Since $B < A$, $B - A < 0$. The expression is negative.

114. (a) Since $C < 0$, $-C > 0$. The expression is positive.

(b) Since $A > C$, $A - C > 0$. The expression is positive.

115. Yes, if a is a negative number, then $-a$ is positive. Thus, $|a| = -a$ if a is negative.

Appendix A.2 Exponents and Radicals

■ You should know the properties of exponents.

(a) $a^1 = a$

(b) $a^0 = 1, a \neq 0$

(c) $a^m a^n = a^{m+n}$

(d) $a^m/a^n = a^{m-n}, a \neq 0$

(e) $a^{-n} = 1/a^n = (1/a)^n, a \neq 0$

(f) $(a^m)^n = a^{mn}$

(g) $(ab)^n = a^n b^n$

(h) $(a/b)^n = a^n/b^n, b \neq 0$

(i) $(a/b)^{-n} = (b/a)^n, a \neq 0, b \neq 0$

(j) $|a^2| = |a|^2 = a^2$

■ You should be able to write numbers in scientific notation, $c \times 10^n$, where $1 \leq c < 10$ and n is an integer.

■ You should be able to use your calculator to evaluate expressions involving exponents.

■ You should know the properties of radicals.

(a) $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m, a > 0$

(b) $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

(c) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}, b \neq 0$

(d) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

(e) $\left(\sqrt[n]{a}\right)^n = a$

(f) For n even, $\sqrt[n]{a^n} = |a|$.
For n odd, $\sqrt[n]{a^n} = a$.

(g) $a^{1/n} = \sqrt[n]{a}$

(h) $a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}, a \geq 0$

■ You should be able to simplify radicals.

(a) All possible factors have been removed from the radical sign.

(b) All fractions have radical-free denominators.

(c) The index for the radical has been reduced as far as possible.

■ You should be able to use your calculator to evaluate radicals.

Vocabulary Check

1. exponent; base **2.** scientific notation **3.** square root
4. principal nth root **5.** index; radicand **6.** simplest form
7. conjugates **8.** rationalizing **9.** power; index

1. $8^5 = 8 \times 8 \times 8 \times 8 \times 8$

2. $(-2)^7 = (-2)(-2)(-2)(-2)(-2)(-2)(-2)$

3. $(4.9)(4.9)(4.9)(4.9)(4.9)(4.9) = 4.9^6$

4. $(-10)(-10)(-10)(-10)(-10) = (-10)^5$

5. (a) $3^2 \cdot 3 = 3^3 = 27$

(b) $3 \cdot 3^3 = 3^4 = 81$

6. (a) $\dfrac{5^5}{5^2} = 5^3 = 125$

(b) $\dfrac{3^2}{3^4} = 3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

7. (a) $(3^3)^0 = 1$

(b) $-3^2 = -9$

8. (a) $(2^3 \cdot 3^2)^2 = 2^{3\cdot 2} \cdot 3^{2\cdot 2}$

$= 2^6 \cdot 3^4 = 64 \cdot 81 = 5184$

(b) $\left(-\dfrac{3}{5}\right)^3\left(\dfrac{5}{3}\right)^2 = (-1)^3\dfrac{3^3}{5^3} \cdot \dfrac{5^2}{3^2} = -1 \cdot 3^{3-2} \cdot 5^{2-3}$

$= -3 \cdot 5^{-1} = -\dfrac{3}{5}$

9. (a) $\dfrac{3 \cdot 4^{-4}}{3^{-4} \cdot 4^{-1}} = 3^{1-(-4)} \cdot 4^{-4-(-1)} = 3^5 \cdot 4^{-3}$

$= \dfrac{3^5}{4^3} = \dfrac{243}{64}$

(b) $32(-2)^{-5} = \dfrac{32}{(-2)^5} = \dfrac{32}{-32} = -1$

10. (a) $\dfrac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}} = 4 \cdot 2^2 \cdot 3^{-2-(-1)} = 4 \cdot 4 \cdot 3^{-1} = \dfrac{16}{3}$

(b) $(-2)^0 = 1$

11. (a) $2^{-1} + 3^{-1} = \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{3}{6} + \dfrac{2}{6} = \dfrac{5}{6}$

(b) $(2^{-1})^{-2} = 2^{(-1)(-2)} = 2^2 = 4$

12. (a) $3^{-1} + 2^{-2} = \dfrac{1}{3} + \dfrac{1}{4} = \dfrac{4}{12} + \dfrac{3}{12} = \dfrac{7}{12}$

(b) $(3^{-2})^2 = 3^{-4} = \dfrac{1}{3^4} = \dfrac{1}{81}$

13. $(-4)^3(5^2) = (-64)(25) = -1600$

14. $(8^{-4})(10^3) \approx 0.244$

15. $\dfrac{3^6}{7^3} = \dfrac{729}{343} \approx 2.125$

16. $\dfrac{4^3}{3^{-4}} = 4^3(3^4) = 5184$

17. When $x = 2$,

$-3x^3 = -3(2)^3 = -24.$

18. When $x = 4$,

$7x^{-2} = 7(4)^{-2} = \dfrac{7}{4^2} = \dfrac{7}{16}.$

19. When $x = 10$,

$6x^0 = 6(10)^0 = 6(1) = 6.$

20. When $x = 3$,

$5(-x)^3 = 5(-3)^3$

$= 5(-27) = -135.$

21. When $x = -3$,

$2x^3 = 2(-3)^3 = 2(-27) = -54.$

22. When $x = -2$,

$-3x^4 = -3(-2)^4$

$= -3(16) = -48.$

23. When $x = -\frac{1}{2}$,

$4x^2 = 4\left(-\frac{1}{2}\right)^2 = 4\left(\frac{1}{4}\right) = 1.$

24. When $x = \frac{1}{3}$,

$5(-x)^3 = 5\left(-\frac{1}{3}\right)^3 = 5\left(-\frac{1}{27}\right) = -\frac{5}{27}.$

25. (a) $(-5z)^3 = (-5)^3z^3 = -125z^3$

(b) $5x^4(x^2) = 5x^{4+2} = 5x^6$

26. (a) $(3x)^2 = 3^2x^2 = 9x^2$

(b) $(4x^3)^0 = 4^0 \cdot x^0 = 1,\ x \neq 0$

27. (a) $6y^2(2y^0)^2 = 6y^2(2 \cdot 1)^2 = 6y^2(4) = 24y^2$

(b) $\dfrac{3x^5}{x^3} = 3x^{5-3} = 3x^2$

28. (a) $(-z)^3(3z^4) = (-1)^3(z^3)3z^4$
$= -1 \cdot 3 \cdot z^{3+4} = -3z^7$

(b) $\dfrac{25y^8}{10y^4} = \dfrac{5}{2}y^{8-4} = \dfrac{5}{2}y^4$

29. (a) $\dfrac{7x^2}{x^3} = 7x^{2-3} = 7x^{-1} = \dfrac{7}{x}$

(b) $\dfrac{12(x+y)^3}{9(x+y)} = \dfrac{4}{3}(x+y)^{3-1} = \dfrac{4}{3}(x+y)^2$

30. (a) $\dfrac{r^4}{r^6} = r^{4-6} = r^{-2} = \dfrac{1}{r^2}$

(b) $\left(\dfrac{4}{y}\right)^3\left(\dfrac{3}{y}\right)^4 = \dfrac{4^3}{y^3} \cdot \dfrac{3^4}{y^4} = \dfrac{64 \cdot 81}{y^{3+4}} = \dfrac{5184}{y^7}$

31. (a) $(x+5)^0 = 1,\ x \neq -5$

(b) $(2x^2)^{-2} = \dfrac{1}{(2x^2)^2} = \dfrac{1}{4x^4}$

32. (a) $(2x^5)^0 = 1,\ x \neq 0$

(b) $(z+2)^{-3}(z+2)^{-1} = (z+2)^{-4} = \dfrac{1}{(z+2)^4}$

33. (a) $(-2x^2)^3\,(4x^3)^{-1} = \dfrac{-8x^6}{4x^3} = -2x^3$

(b) $\left(\dfrac{x}{10}\right)^{-1} = \dfrac{10}{x}$

34. (a) $(4y^{-2})(8y^4) = (4)(8)(y^{4-2}) = 32y^2$

(b) $\left(\dfrac{x^{-3}y^4}{5}\right)^{-3} = \left(\dfrac{5x^3}{y^4}\right)^3 = \dfrac{125x^9}{y^{12}}$

35. (a) $3^n \cdot 3^{2n} = 3^{n+2n} = 3^{3n}$

(b) $\left(\dfrac{a^{-2}}{b^{-2}}\right)\left(\dfrac{b}{a}\right)^3 = \left(\dfrac{b^2}{a^2}\right)\left(\dfrac{b^3}{a^3}\right) = \dfrac{b^5}{a^5}$

36. (a) $\dfrac{x^2 \cdot x^n}{x^3 \cdot x^n} = \dfrac{x^{2+n}}{x^{3+n}} = x^{2+n-3-n} = x^{-1} = \dfrac{1}{x}$

(b) $\left(\dfrac{a^{-3}}{b^{-3}}\right)\left(\dfrac{a}{b}\right)^3 = \dfrac{a^{-3}}{b^{-3}} \cdot \dfrac{a^3}{b^3} = \dfrac{a^{-3+3}}{b^{-3+3}} = \dfrac{a^0}{b^0} = 1$

37. 57,300,000 $= 5.73 \times 10^7$ square miles

38. 9,460,000,000,000 $= 9.460 \times 10^{12}$ kilometers

39. 0.0000899 $= 8.99 \times 10^{-5}$ gram per cubic centimeter

40. 0.00003937 $= 3.937 \times 10^{-5}$ inch

41. $4.568 \times 10^9 =$ 4,568,000,000 ounces

42. $1.5 \times 10^7 =$ 15,000,000 degrees Celsius

43. $1.6022 \times 10^{-19} =$ 0.00000000000000000016022 coulomb

44. $9.0 \times 10^{-5} =$ 0.00009 meter

45. (a) $\sqrt{25 \times 10^8} = 5 \times 10^4 = 50{,}000$

(b) $\sqrt[3]{8 \times 10^{15}} = 2 \times 10^5 = 200{,}000$

46. (a) $(1.2 \times 10^7)(5 \times 10^{-3}) = 6.0 \times 10^4 = 60{,}000$

(b) $\dfrac{(6.0 \times 10^8)}{(3.0 \times 10^{-3})} = 2.0 \times 10^{11}$

47. (a) $750\left(1 + \dfrac{0.11}{365}\right)^{800} \approx 954.448$

(b) $\dfrac{67{,}000{,}000 + 93{,}000{,}000}{0.0052} = 30{,}769{,}230{,}769.2$
$\approx 3.077 \times 10^{10}$

48. (a) $(9.3 \times 10^6)^3(6.1 \times 10^{-4}) \approx 4.907 \times 10^{17}$

(b) $\dfrac{(2.414 \times 10^4)^6}{(1.68 \times 10^5)^5} \approx 1.479$

49. (a) $\sqrt{4.5 \times 10^9} \approx 67{,}082.039$

(b) $\sqrt[3]{6.3 \times 10^4} \approx 39.791$

50. (a) $(2.65 \times 10^{-4})^{1/3} \approx 0.064$

(b) $\sqrt{9 \times 10^{-4}} = 0.03$

51. (a) $\sqrt{9} = 3$

(b) $\sqrt[3]{\dfrac{27}{8}} = \dfrac{\sqrt[3]{27}}{\sqrt[3]{8}} = \dfrac{3}{2}$

52. (a) $27^{1/3} = \sqrt[3]{27} = 3$

(b) $36^{3/2} = 216$

53. (a) $32^{-3/5} = \dfrac{1}{32^{3/5}} = \dfrac{1}{\left(\sqrt[5]{32}\right)^3} = \dfrac{1}{(2)^3} = \dfrac{1}{8}$

(b) $\left(\dfrac{16}{81}\right)^{-3/4} = \left(\dfrac{81}{16}\right)^{3/4} = \left(\sqrt[4]{\dfrac{81}{16}}\right)^3 = \left(\dfrac{3}{2}\right)^3 = \dfrac{27}{8}$

54. (a) $100^{-3/2} = \left(\sqrt{100}\right)^{-3} = 10^{-3} = \dfrac{1}{1000}$

(b) $\left(\dfrac{9}{4}\right)^{-1/2} = \left(\dfrac{4}{9}\right)^{1/2} = \dfrac{\sqrt{4}}{\sqrt{9}} = \dfrac{2}{3}$

55. (a) $\left(-\dfrac{1}{64}\right)^{-1/3} = (-64)^{1/3} = \sqrt[3]{-64} = -4$

(b) $\left(\dfrac{1}{\sqrt{32}}\right)^{-2/5} = \left(\sqrt{32}\right)^{2/5} = \sqrt[5]{\left(\sqrt{32}\right)^2} = \sqrt[5]{32} = 2$

56. (a) $\left(-\dfrac{125}{27}\right)^{-1/3} = \left(-\dfrac{27}{125}\right)^{1/3} = \dfrac{\sqrt[3]{-27}}{\sqrt[3]{125}} = \dfrac{-3}{5} = -\dfrac{3}{5}$

(b) $-\left(\dfrac{1}{125}\right)^{-4/3} = -(125)^{4/3}$

$= -(125^{1/3})^4 = -\left(\sqrt[3]{125}\right)^4$

$= -(5)^4 = -625$

57. (a) $\sqrt{57} \approx 7.550$

(b) $\sqrt[5]{-27^3} = (-27)^{3/5}$

≈ -7.225

58. (a) $\sqrt[3]{45^2} \approx 12.651$

(b) $\sqrt[6]{125} \approx 2.236$

59. (a) $(-12.4)^{-1.8} \approx -0.011$

(b) $\left(5\sqrt{3}\right)^{-2.5} \approx 0.005$

60. (a) $\dfrac{7 - (4.1)^{-3.2}}{2} \approx 3.495$

(b) $\left(\dfrac{13}{3}\right)^{-3/2} - \left(-\dfrac{3}{2}\right)^{13/3} \approx 5.906$

61. (a) $\left(\sqrt[3]{4}\right)^3 = 4^{3/3} = 4^1 = 4$

(b) $\sqrt[5]{96x^5} = \sqrt[5]{32x^5 \cdot 3}$

$= 2x\sqrt[5]{3}$

$= 2 \cdot 3^{1/5} \cdot x$

62. (a) $\sqrt{12} \cdot \sqrt{3} = \sqrt{36} = 6$

(b) $\sqrt[4]{(3x^2)^4} = \sqrt[4]{3^4x^8}$

$= 3x^2$

63. (a) $\sqrt{8} = \sqrt{4 \cdot 2}$

$= \sqrt{4}\sqrt{2} = 2\sqrt{2}$

(b) $\sqrt[3]{24} = \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8}\sqrt[3]{3} = 2\sqrt[3]{3}$

64. (a) $\sqrt[3]{\dfrac{16}{27}} = \dfrac{\sqrt[3]{2^3 \cdot 2}}{\sqrt[3]{3^3}} = \dfrac{2\sqrt[3]{2}}{3}$

(b) $\sqrt{\dfrac{75}{4}} = \dfrac{\sqrt{5^2 \cdot 3}}{\sqrt{2^2}} = \dfrac{5\sqrt{3}}{2}$

65. (a) $\sqrt{72x^3} = \sqrt{36x^2 \cdot 2x}$

$= 6x\sqrt{2x}$

(b) $\sqrt{\dfrac{18^2}{z^3}} = \dfrac{\sqrt{18^2}}{\sqrt{z^2 \cdot z}} = \dfrac{18}{z\sqrt{z}}$

66. (a) $\sqrt{54xy^4} = \sqrt{6 \cdot 3^2 \cdot x \cdot (y^2)^2}$

$= 3y^2\sqrt{6x}$

(b) $\sqrt{\dfrac{32a^4}{b^2}} = \dfrac{\sqrt{(2^2)^2 \cdot 2 \cdot (a^2)^2}}{\sqrt{b^2}}$

$= \dfrac{4a^2\sqrt{2}}{|b|}$

67. (a) $\sqrt[3]{16x^5} = \sqrt[3]{8x^3 \cdot 2x^2}$

$= 2x\sqrt[3]{2x^2}$

(b) $\sqrt{75x^2y^{-4}} = \sqrt{\dfrac{75x^2}{y^4}}$

$= \dfrac{\sqrt{25x^2 \cdot 3}}{\sqrt{y^4}}$

$= \dfrac{5|x|\sqrt{3}}{y^2}$

68. (a) $\sqrt[4]{3x^4y^2} = (3x^4y^2)^{1/4}$

$= 3^{1/4}|x|y^{1/2}$

$= |x|\sqrt[4]{3}\sqrt{y}$

(b) $\sqrt[5]{160x^8z^4} = (160x^8z^4)^{1/5}$

$= (2^5 \cdot 5x^5 \cdot x^3 \cdot z^4)^{1/5}$

$= 2x(5x^3z^4)^{1/5}$

$= 2x\sqrt[5]{5x^3z^4}$

69. (a) $2\sqrt{50} + 12\sqrt{8} = 2\sqrt{25 \cdot 2} + 12\sqrt{4 \cdot 2} = 2(5\sqrt{2}) + 12(2\sqrt{2}) = 10\sqrt{2} + 24\sqrt{2} = 34\sqrt{2}$

(b) $10\sqrt{32} - 6\sqrt{18} = 10\sqrt{16 \cdot 2} - 6\sqrt{9 \cdot 2} = 10(4\sqrt{2}) - 6(3\sqrt{2}) = 40\sqrt{2} - 18\sqrt{2} = 22\sqrt{2}$

70. (a) $4\sqrt{27} - \sqrt{75} = 4\sqrt{3^2 \cdot 3} - \sqrt{5^2 \cdot 3}$
$= 4 \cdot 3\sqrt{3} - 5\sqrt{3}$
$= 12\sqrt{3} - 5\sqrt{3}$
$= 7\sqrt{3}$

(b) $\sqrt[3]{16} + 3\sqrt[3]{54} = \sqrt[3]{2 \cdot 2^3} + 3\sqrt[3]{2 \cdot 3^3}$
$= 2\sqrt[3]{2} + 3 \cdot 3\sqrt[3]{2}$
$= 2\sqrt[3]{2} + 9\sqrt[3]{2}$
$= 11\sqrt[3]{2}$

71. (a) $5\sqrt{x} - 3\sqrt{x} = 2\sqrt{x}$

(b) $-2\sqrt{9y} + 10\sqrt{y} = -2(3\sqrt{y}) + 10\sqrt{y}$
$= -6\sqrt{y} + 10\sqrt{y} = 4\sqrt{y}$

72. (a) $8\sqrt{49x} - 14\sqrt{100x} = 8\sqrt{7^2 \cdot x} - 14\sqrt{10^2 \cdot x}$
$= 8 \cdot 7\sqrt{x} - 14 \cdot 10\sqrt{x} = 56\sqrt{x} - 140\sqrt{x}$
$= -84\sqrt{x}$

(b) $-3\sqrt{48x^2} + 7\sqrt{75x^2} = -3\sqrt{3 \cdot 4^2 \cdot x^2} + 7\sqrt{3 \cdot 5^2 \cdot x^2}$
$= -3 \cdot 4|x|\sqrt{3} + 7 \cdot 5|x|\sqrt{3}$
$= -12|x|\sqrt{3} + 35|x|\sqrt{3} = 23|x|\sqrt{3}$

73. (a) $3\sqrt{x+1} + 10\sqrt{x+1} = 13\sqrt{x+1}$

(b) $7\sqrt{80x} - 2\sqrt{125x} = 7\sqrt{16 \cdot 5x} - 2\sqrt{25 \cdot 5x} = 7(4\sqrt{5x}) - 2(5\sqrt{5x}) = 28\sqrt{5x} - 10\sqrt{5x} = 18\sqrt{5x}$

74. (a) $-\sqrt{x^3 - 7} + 5\sqrt{x^3 - 7} = 4\sqrt{x^3 - 7}$

(b) $11\sqrt{245x^3} - 9\sqrt{45x^3} = 11\sqrt{5 \cdot 7^2 \cdot x \cdot x^2} - 9\sqrt{5 \cdot 3^2 \cdot x \cdot x^2}$
$= 11 \cdot 7x\sqrt{5x} - 9 \cdot 3x\sqrt{5x}$
$= 77x\sqrt{5x} - 27x\sqrt{5x}$
$= 50x\sqrt{5x}$

75. $\sqrt{5} + \sqrt{3} \approx 3.968$ and
$\sqrt{5+3} = \sqrt{8} \approx 2.828$
Thus, $\sqrt{5} + \sqrt{3} > \sqrt{5+3}$.

76. $\sqrt{\dfrac{3}{11}} = \dfrac{\sqrt{3}}{\sqrt{11}}$

77. $\sqrt{3^2 + 2^2} = \sqrt{9+4}$
$= \sqrt{13} \approx 3.606$
Thus, $5 > \sqrt{3^2 + 2^2}$.

78. $\sqrt{3^2 + 4^2} = \sqrt{9 + 16}$
$= \sqrt{25} = 5$
Thus, $5 = \sqrt{3^2 + 4^2}$.

79. $\dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

80. $\dfrac{5}{\sqrt{10}} = \dfrac{5}{\sqrt{10}} \cdot \dfrac{\sqrt{10}}{\sqrt{10}}$
$= \dfrac{5\sqrt{10}}{10} = \dfrac{\sqrt{10}}{2}$

81. $\dfrac{2}{5 - \sqrt{3}} = \dfrac{2}{5 - \sqrt{3}} \cdot \dfrac{5 + \sqrt{3}}{5 + \sqrt{3}} = \dfrac{2(5 + \sqrt{3})}{5^2 - (\sqrt{3})^2} = \dfrac{2(5 + \sqrt{3})}{25 - 3} = \dfrac{2(5 + \sqrt{3})}{22} = \dfrac{5 + \sqrt{3}}{11}$

82. $\dfrac{3}{\sqrt{5} + \sqrt{6}} = \dfrac{3}{\sqrt{5} + \sqrt{6}} \cdot \dfrac{\sqrt{5} - \sqrt{6}}{\sqrt{5} - \sqrt{6}} = \dfrac{3(\sqrt{5} - \sqrt{6})}{5 - 6} = \dfrac{3(\sqrt{5} - \sqrt{6})}{-1} = -3(\sqrt{5} - \sqrt{6}) = 3(\sqrt{6} - \sqrt{5})$

83. $\dfrac{\sqrt{8}}{2} = \dfrac{\sqrt{4 \cdot 2}}{2} = \dfrac{2\sqrt{2}}{2} = \dfrac{\sqrt{2}}{1} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2}{\sqrt{2}}$

84. $\dfrac{\sqrt{2}}{3} = \dfrac{\sqrt{2}}{3} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2}{3\sqrt{2}}$

85. $\dfrac{\sqrt{5} + \sqrt{3}}{3} = \dfrac{\sqrt{5} + \sqrt{3}}{3} \cdot \dfrac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}} = \dfrac{5 - 3}{3\left(\sqrt{5} - \sqrt{3}\right)} = \dfrac{2}{3\left(\sqrt{5} - \sqrt{3}\right)}$

86. $\dfrac{\sqrt{7} - 3}{4} = \dfrac{\sqrt{7} - 3}{4} \cdot \dfrac{\sqrt{7} + 3}{\sqrt{7} + 3} = \dfrac{7 - 9}{4\left(\sqrt{7} + 3\right)} = \dfrac{-2}{4\left(\sqrt{7} + 3\right)} = -\dfrac{1}{2\left(\sqrt{7} + 3\right)}$

	Radical Form	*Rational Exponent Form*
87.	$\sqrt{9} = 3$, Given	$9^{1/2} = 3$, Answer
88.	$\sqrt[3]{64} = 4$, Given	$64^{1/3} = 4$, Answer
89.	$\sqrt[5]{32} = 2$, Answer	$32^{1/5} = 2$, Given
90.	$-\sqrt{144} = -12$, Answer	$-(144^{1/2}) = -12$, Given
91.	$\sqrt[3]{-216} = -6$, Given	$(-216)^{1/3} = -6$, Answer
92.	$\sqrt[5]{-243} = -3$, Answer	$(-243)^{1/5} = -3$, Given
93.	$\sqrt[4]{81^3} = 27$, Given	$81^{3/4} = 27$, Answer
94.	$\sqrt[4]{16^5} = 32$, Answer	$16^{5/4} = 32$, Given

95. $\dfrac{(2x^2)^{3/2}}{2^{1/2}x^4} = \dfrac{2^{3/2}(x^2)^{3/2}}{2^{1/2}x^4}$

$= \dfrac{2^{3/2}x^3}{2^{1/2}x^4} = 2^{3/2-1/2}x^{3-4} = 2^1x^{-1} = \dfrac{2}{x}$

96. $\dfrac{x^{4/3}y^{2/3}}{(xy)^{1/3}} = \dfrac{x^{4/3}y^{2/3}}{x^{1/3}y^{1/3}} = x^{3/3}y^{1/3} = xy^{1/3}$

97. $\dfrac{x^{-3} \cdot x^{1/2}}{x^{3/2} \cdot x^{-1}} = \dfrac{x^{1/2} \cdot x^1}{x^{3/2} \cdot x^3}$

$= x^{1/2+1-3/2-3} = x^{-3} = \dfrac{1}{x^3}, \ x > 0$

98. $\dfrac{5^{-1/2} \cdot 5x^{5/2}}{(5x)^{3/2}} = \dfrac{5^{-1/2} \cdot 5x^{5/2}}{5^{3/2}x^{3/2}}$

$= \dfrac{5^{1/2}x^{5/2}}{5^{3/2}x^{3/2}} = 5^{-1}x = \dfrac{x}{5}, \ x > 0$

99. (a) $\sqrt[4]{3^2} = 3^{2/4} = 3^{1/2} = \sqrt{3}$

(b) $\sqrt[6]{(x+1)^4} = (x+1)^{4/6} = (x+1)^{2/3} = \sqrt[3]{(x+1)^2}$

100. (a) $\sqrt[6]{x^3} = x^{3/6} = x^{1/2} = \sqrt{x}$

(b) $\sqrt[4]{(3x^2)^4} = 3x^2$

101. (a) $\sqrt{\sqrt{32}} = (32^{1/2})^{1/2}$

$= 32^{1/4} = \sqrt[4]{32} = \sqrt[4]{16 \cdot 2} = 2\sqrt[4]{2}$

(b) $\sqrt{\sqrt[4]{2x}} = ((2x)^{1/4})^{1/2} = (2x)^{1/8} = \sqrt[8]{2x}$

102. (a) $\sqrt{\sqrt{243(x+1)}} = [(243(x+1)]^{1/2})^{1/2}$

$= (243(x+1))^{1/4}$

$= \sqrt[4]{243(x+1)}$

$= \sqrt[4]{3 \cdot 81(x+1)}$

$= 3\sqrt[4]{3(x+1)}$

(b) $\sqrt{\sqrt[3]{10a^7b}} = ((10a^7b)^{1/3})^{1/2}$

$= (10a^7b)^{1/6}$

$= \sqrt[6]{10a \cdot a^6 \cdot b}$

$= a\sqrt[6]{10ab}$

103. $T = 2\pi\sqrt{\dfrac{2}{32}}$

$= 2\pi\sqrt{\dfrac{1}{16}}$

$= 2\pi\left(\dfrac{1}{4}\right)$

$= \dfrac{\pi}{2} \approx 1.57$ seconds

104. Size $= 0.03\sqrt{v}$; For $v = \dfrac{3}{4}$:

$\text{Size} = 0.03\sqrt{\dfrac{3}{4}}$

$= 0.03 \cdot \dfrac{\sqrt{3}}{\sqrt{4}}$

$= 0.03\dfrac{\sqrt{3}}{2}$

≈ 0.026 inch

105. $t = 0.03[12^{5/2} - (12 - h)^{5/2}]$, $0 \le h \le 12$

(a)

h (in centimeters)	t (in seconds)
0	0
1	2.93
2	5.48
3	7.67
4	9.53
5	11.08
6	12.32
7	13.29
8	14.00
9	14.50
10	14.80
11	14.93
12	14.96

(b) As h approaches 12, t approaches $0.03(12^{5/2}) = 8.64\sqrt{3} \approx 14.96$ seconds.

106. $\text{Time} = \dfrac{\text{Distance}}{\text{Rate}} = \dfrac{93{,}000{,}000 \text{ miles}}{11{,}180{,}000 \text{ miles per minute}}$

≈ 8.32 minutes, or 8 minutes 19.1 seconds

107. True. When dividing variables, you subtract exponents.

108. False. When a power is raised to a power, you multiply the exponents: $(a^n)^k = a^{nk}$.

109. $1 = \dfrac{a^m}{a^m} = a^{m-m} = a^0, a \ne 0$

110. (a) 3 is also raised to the negative one power so, $(3x)^{-1} = \dfrac{1}{3x}$.

(b) When two powers have the same base, the exponents are added, $y^3 \cdot y^2 = y^5$.

(c) When a power is raised to a power, exponents are multiplied, $(a^2b^3)^4 = a^8b^{12}$.

(d) The square of a binomial contains a cross product term, $(a + b)^2 = a^2 + 2ab + b^2$.

(e) If $x < 0$, then $\sqrt{4x^2} > 0$ but $2x < 0$, $\sqrt{4x^2} = 2|x|$.

(f) Radicals can only be added together if they have the same radicand and index: $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$.

111. When any positive integer is squared, the units digit is 0, 1, 4, 5, 6, or 9. Therefore, $\sqrt{5233}$ is not an integer.

112. $\left(\frac{2}{\sqrt{5}}\right)^2 = \frac{4}{5} = 0.8$

$\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5} = 0.8944$

Since $0.8 \neq 0.8944$, $\left(\frac{2}{\sqrt{5}}\right)^2 \neq \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

and squaring is not equivalent to rationalizing the denominator.

Appendix A.3 Polynomials and Factoring

- Given a polynomial in x, $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where $a_n \neq 0$, and n is a nonnegative integer, you should be able to identify the following.
  - (a) Degree: n
  - (b) Terms: $a_n x^n, a_{n-1}x^{n-1}, \ldots, a_1 x, a_0$
  - (c) Coefficients: $a_n, a_{n-1}, \ldots, a_1, a_0$
  - (d) Leading coefficient: a_n
  - (e) Constant term: a_0
- You should be able to add and subtract polynomials.
- You should be able to multiply polynomials by the Distributive Properties.
- You should be able to multiply two binomials by the FOIL Method.
- You should know the special binomial products.
  - (a) $(u + v)(u - v) = u^2 - v^2$
  - (b) $(u \pm v)^2 = u^2 \pm 2uv + v^2$
  - (c) $(u \pm v)^3 = u^3 \pm 3u^2v + 3uv^2 \pm v^3$
- You should be able to factor out all common factors, the first step in factoring.
- You should be able to factor the following special polynomial forms.
  - (a) $u^2 - v^2 = (u + v)(u - v)$
  - (b) $u^2 \pm 2uv + v^2 = (u \pm v)^2$
  - (c) $u^3 \pm v^3 = (u \pm v)(u^2 \mp uv + v^2)$
- You should be able to factor by grouping.
- You should be able to factor some trinomials by grouping.

Vocabulary Check

1. n; a_n; a_0

2. descending

3. monomial; binomial; trinomial

4. like terms

5. First terms; Outer terms; Inner terms; Last terms

6. factoring

7. completely factored

1. (d) 12 is a polynomial of degree zero.

2. (e) $-3x^5 + 2x^3 + x$ is a polynomial of degree five.

3. (b) $1 - 2x^3 = -2x^3 + 1$ is a binomial with leading coefficient -2.

4. (a) $3x^2$ is a monomial of positive degree.

5. (f) $\frac{2}{3}x^4 + x^2 + 10$ is a trinomial with leading coefficient $\frac{2}{3}$.

6. (c) $x^3 + 3x^2 + 3x + 1$ is a third-degree polynomial with leading coefficient 1.

7. $-2x^3$; $-2x^3 + 5$; $-2x^3 + 4x^2 - 3x + 20$, etc. (Answers will vary.)

8. $6x^5 + 3x + 1$ (Answers will vary.)

9. $-15x^4 + 1$; $-3x^4 + 7x^2$; $-5x^4 - 6x$, etc. (Answers will vary.)

10. $20x^3 + 5$ (Answers will vary.)

11. (a) Standard form: $-\frac{1}{2}x^5 + 14x$

(b) Degree: 5

Leading coefficient: $-\frac{1}{2}$

(c) Binomial

12. (a) Standard form: $2x^2 - x + 1$

(b) Degree: 2

Leading coefficient: 2

(c) Trinomial

13. (a) Standard form: $-3x^4 + 2x^2 - 5$

(b) Degree: 4

Leading coefficient: -3

(c) Trinomial

14. (a) Standard form: $7x$

(b) Degree: 1

Leading coefficient: 7

(c) Monomial

15. (a) Standard form: $x^5 - 1$

(b) Degree: 5

Leading coefficient: 1

(c) Binomial

16. (a) Standard form: $25y^2 - y + 1$

(b) Degree: 2

Leading coefficient: 25

(c) Trinomial

17. (a) Standard form: 3

(b) Degree: 0

Leading coefficient: 3

(c) Monomial

18. (a) Standard form: $t^2 + 9$

(b) Degree: 2

Leading coefficient: 1

(c) Binomial

19. (a) Standard form: $-4x^5 + 6x^4 + 1$

(b) Degree: 5

Leading coefficient: -4

(c) Trinomial

20. (a) Standard form: $2x + 3$

(b) Degree: 1

Leading coefficient: 2

(c) Binomial

21. (a) Standard form: $4x^3y$

(b) Degree: 4 (add the exponents on x and y)

Leading coefficient: 4

(c) Monomial

22. (a) Standard form: $-x^5y + 2x^2y^2 + xy^4$

(b) Degree: 6

Leading coefficient: -1

(c) Trinomial

23. $2x - 3x^3 + 8$ *is* a polynomial.
Standard form: $-3x^3 + 2x + 8$

24. $2x^3 + x - 3x^{-1}$ is *not* a polynomial because it includes a term with a negative exponent.

25. $\dfrac{3x + 4}{x} = 3 + \dfrac{4}{x} = 3 + 4x^{-1}$ is *not* a polynomial because it includes a term with a negative exponent.

26. $\dfrac{x^2 + 2x - 3}{2}$ *is* a polynomial.

Standard form: $\dfrac{1}{2}x^2 + x - \dfrac{3}{2}$

27. $y^2 - y^4 + y^3$ *is* a polynomial.
Standard form: $-y^4 + y^3 + y^2$

28. $\sqrt{y^2 - y^4}$ is *not* a polynomial because of the square root.

29. $(6x + 5) - (8x + 15) = 6x + 5 - 8x - 15$
$= (6x - 8x) + (5 - 15)$
$= -2x - 10$

30. $(2x^2 + 1) - (x^2 - 2x + 1) = 2x^2 + 1 - x^2 + 2x - 1$
$= (2x^2 - x^2) + 2x + (1 - 1)$
$= x^2 + 2x$

31. $-(x^3 - 2) + (4x^3 - 2x) = -x^3 + 2 + 4x^3 - 2x$
$= (4x^3 - x^3) - 2x + 2$
$= 3x^3 - 2x + 2$

32. $-(5x^2 - 1) - (-3x^2 + 5) = -5x^2 + 1 + 3x^2 - 5$
$= (-5x^2 + 3x^2) + (1 - 5)$
$= -2x^2 - 4$

33. $(15x^2 - 6) - (-8.3x^3 - 14.7x^2 - 17) = 15x^2 - 6 + 8.3x^3 + 14.7x^2 + 17$
$= 8.3x^3 + (15x^2 + 14.7x^2) + (-6 + 17)$
$= 8.3x^3 + 29.7x^2 + 11$

34. $(15.2x^4 - 18x - 19.1) - (13.9x^4 - 9.6x + 15) = 15.2x^4 - 18x - 19.1 - 13.9x^4 + 9.6x - 15$
$= (15.2x^4 - 13.9x^4) + (-18x + 9.6x) + (-19.1 - 15)$
$= 1.3x^4 - 8.4x - 34.1$

35. $5z - [3z - (10z + 8)] = 5z - (3z - 10z - 8)$
$= 5z - 3z + 10z + 8$
$= (5z - 3z + 10z) + 8$
$= 12z + 8$

36. $(y^3 + 1) - [(y^2 + 1) + (3y - 7)] = y^3 + 1 - (y^2 + 1) - (3y - 7)$
$= y^3 + 1 - y^2 - 1 - 3y + 7$
$= y^3 - y^2 - 3y + (1 - 1 + 7)$
$= y^3 - y^2 - 3y + 7$

37. $3x(x^2 - 2x + 1) = 3x(x^2) + 3x(-2x) + 3x(1)$
$= 3x^3 - 6x^2 + 3x$

38. $y^2(4y^2 + 2y - 3) = y^2(4y^2) + y^2(2y) + y^2(-3)$
$= 4y^4 + 2y^3 - 3y^2$

39. $-5z(3z - 1) = -5z(3z) + (-5z)(-1)$
$= -15z^2 + 5z$

40. $(-3x)(5x + 2) = -3x(5x) + (-3x)(2)$
$= -15x^2 - 6x$

41. $(1 - x^3)(4x) = 1(4x) - x^3(4x)$
$= 4x - 4x^4$
$= -4x^4 + 4x$

42. $-4x(3 - x^3) = -4x(3) + (-4x)(-x^3)$
$= -12x + 4x^4$
$= 4x^4 - 12x$

43. $(2.5x^2 + 3)(3x) = (2.5x^2)(3x) + (3)(3x)$
$= 7.5x^3 + 9x$

44. $(2 - 3.5y)(2y^3) = 2(2y^3) + (-3.5y)(2y^3)$
$= 4y^3 - 7y^4 = -7y^4 + 4y^3$

45. $-4x\left(\frac{1}{8}x + 3\right) = (-4x)\left(\frac{1}{8}x\right) + (-4x)(3)$
$= -\frac{1}{2}x^2 - 12x$

46. $2y\left(4 - \frac{7}{8}y\right) = 2y(4) + 2y\left(-\frac{7}{8}y\right)$
$= 8y - \frac{7}{4}y^2$
$= -\frac{7}{4}y^2 + 8y$

47. $(x + 3)(x + 4) = x^2 + 4x + 3x + 12$ FOIL
$= x^2 + 7x + 12$

48. $(x - 5)(x + 10) = x^2 + 10x - 5x - 50$ FOIL
$= x^2 + 5x - 50$

49. $(3x - 5)(2x + 1) = 6x^2 + 3x - 10x - 5$ FOIL
$= 6x^2 - 7x - 5$

50. $(7x - 2)(4x - 3) = 28x^2 - 21x - 8x + 6$ FOIL
$= 28x^2 - 29x + 6$

51. Multiply:
$$\begin{array}{r} x^2 - x + 1 \\ \underline{x^2 + x + 1} \\ x^4 - x^3 + x^2 \\ x^3 - x^2 + x \\ \underline{\qquad\quad x^2 - x + 1} \\ x^4 - 0x^3 + x^2 + 0x + 1 \end{array} = x^4 + x^2 + 1$$

52. Multiply:
$$\begin{array}{r} x^2 + 3x - 2 \\ \underline{x^2 - 3x - 2} \\ x^4 + 3x^3 - 2x^2 \\ -3x^3 - 9x^2 + 6x \\ \underline{\qquad - 2x^2 - 6x + 4} \\ x^4 + 0x^3 - 13x^2 + 0x + 4 \end{array} = x^4 - 13x^2 + 4$$

53. $(x + 10)(x - 10) = x^2 - 10^2 = x^2 - 100$

54. $(2x + 3)(2x - 3) = (2x)^2 - 3^2 = 4x^2 - 9$

55. $(x + 2y)(x - 2y) = x^2 - (2y)^2 = x^2 - 4y^2$

56. $(2x + 3y)(2x - 3y) = (2x)^2 - (3y)^2 = 4x^2 - 9y^2$

57. $(2x + 3)^2 = (2x)^2 + 2(2x)(3) + 3^2$
$= 4x^2 + 12x + 9$

58. $(4x + 5)^2 = (4x)^2 + 2(4x)(5) + 5^2$
$= 16x^2 + 40x + 25$

59. $(2x - 5y)^2 = (2x)^2 - 2(2x)(5y) + (5y)^2$
$= 4x^2 - 20xy + 25y^2$

60. $(5 - 8x)^2 = 5^2 + (2)(5)(-8x) + (-8x)^2$
$= 25 - 80x + 64x^2$

61. $(x + 1)^3 = x^3 + 3x^2(1) + 3x(1^2) + 1^3$
$= x^3 + 3x^2 + 3x + 1$

62. $(x - 2)^3 = x^3 - 3x^2(2) + 3x(2)^2 - 2^3$
$= x^3 - 6x^2 + 12x - 8$

63. $(2x - y)^3 = (2x)^3 - 3(2x)^2y + 3(2x)y^2 - y^3$
$= 8x^3 - 12x^2y + 6xy^2 - y^3$

64. $(3x + 2y)^3 = (3x)^3 + 3(3x)^2(2y) + 3(3x)(2y)^2 + (2y)^3$
$= 27x^3 + 54x^2y + 36xy^2 + 8y^3$

65. $(4x^3 - 3)^2 = (4x^3)^2 - 2(4x^3)(3) + (3)^2$
$= 16x^6 - 24x^3 + 9$

66. $(8x + 3)^2 = (8x)^2 + 2(8x)(3) + 3^2$
$= 64x^2 + 48x + 9$

67. $[(m - 3) + n][(m - 3) - n] = (m - 3)^2 - n^2$
$= m^2 - 6m + 9 - n^2$
$= m^2 - n^2 - 6m + 9$

68. $[(x + y) + 1][(x + y) - 1] = (x + y)^2 - 1^2$
$= x^2 + 2xy + y^2 - 1$

69. $[(x - 3) + y]^2 = (x - 3)^2 + 2y(x - 3) + y^2$
$= x^2 - 6x + 9 + 2xy - 6y + y^2$
$= x^2 + 2xy + y^2 - 6x - 6y + 9$

70. $[(x + 1) - y]^2 = (x + 1)^2 + 2(x + 1)(-y) + (-y)^2$
$= x^2 + 2x + 1 - 2xy - 2y + y^2$
$= x^2 - 2xy + y^2 + 2x - 2y + 1$

71. $(2r^2 - 5)(2r^2 + 5) = (2r^2)^2 - 5^2 = 4r^4 - 25$

72. $(3a^3 - 4b^2)(3a^3 + 4b^2) = (3a^3)^2 - (4b^2)^2$
$= 9a^6 - 16b^4$

73. $\left(\frac{1}{2}x - 3\right)^2 = \left(\frac{1}{2}x\right)^2 - 2\left(\frac{1}{2}x\right)(3) + 3^2$
$= \frac{1}{4}x^2 - 3x + 9$

74. $\left(\frac{2}{3}t + 5\right)^2 = \left(\frac{2}{3}t\right)^2 + 2\left(\frac{2}{3}t\right)(5) + (5)^2$
$= \frac{4}{9}t^2 + \frac{20}{3}t + 25$

75. $\left(\frac{1}{3}x - 2\right)\left(\frac{1}{3}x + 2\right) = \left(\frac{1}{3}x\right)^2 - (2)^2$
$= \frac{1}{9}x^2 - 4$

76. $\left(2x + \frac{1}{5}\right)\left(2x - \frac{1}{5}\right) = (2x)^2 - \left(\frac{1}{5}\right)^2$
$= 4x^2 - \frac{1}{25}$

77. $(1.2x + 3)^2 = (1.2x)^2 + 2(1.2x)(3) + 3^2$
$= 1.44x^2 + 7.2x + 9$

78. $(1.5y - 3)^2 = (1.5y)^2 + 2(1.5y)(-3) + (-3)^2$
$= 2.25y^2 - 9y + 9$

79. $(1.5x - 4)(1.5x + 4) = (1.5x)^2 - 4^2$
$= 2.25x^2 - 16$

80. $(2.5y + 3)(2.5y - 3) = (2.5y)^2 - (3)^2$
$= 6.25y^2 - 9$

81. $5x(x + 1) - 3x(x + 1) = 2x(x + 1)$
$= 2x^2 + 2x$

82. $(2x - 1)(x + 3) + 3(x + 3) = (2x + 2)(x + 3) = 2x^2 + 6x + 2x + 6$ FOIL
$= 2x^2 + 8x + 6$

83. $(u + 2)(u - 2)(u^2 + 4) = (u^2 - 4)(u^2 + 4)$
$= u^4 - 16$

84. $(x + y)(x - y)(x^2 + y^2) = (x^2 - y^2)(x^2 + y^2)$
$= (x^2)^2 - (y^2)^2 = x^4 - y^4$

85. $\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y}\right) = \left(\sqrt{x}\right)^2 - \left(\sqrt{y}\right)^2$
$= x - y$

86. $\left(5 + \sqrt{x}\right)\left(5 - \sqrt{x}\right) = (5)^2 - \left(\sqrt{x}\right)^2$
$= 25 - x$

87. $\left(x - \sqrt{5}\right)^2 = x^2 - 2(x)\left(\sqrt{5}\right) + \left(\sqrt{5}\right)^2$
$= x^2 - 2\sqrt{5}x + 5$

88. $\left(x + \sqrt{3}\right)^2 = x^2 + 2x\sqrt{3} + \left(\sqrt{3}\right)^2$
$= x^2 + 2\sqrt{3}x + 3$

89. $3x + 6 = 3(x + 2)$

90. $5y - 30 = 5(y - 6)$

91. $2x^3 - 6x = 2x(x^2 - 3)$

92. $4x^3 - 6x^2 + 12x = 2x(2x^2 - 3x + 6)$

93. $x(x - 1) + 6(x - 1) = (x - 1)(x + 6)$

94. $3x(x + 2) - 4(x + 2) = (x + 2)(3x - 4)$

95. $(x + 3)^2 - 4(x + 3) = (x + 3)[(x + 3) - 4]$
$= (x + 3)(x - 1)$

96. $(3x - 1)^2 + (3x - 1) = (3x - 1 + 1)(3x - 1)$
$= 3x(3x - 1)$

97. $\frac{1}{2}x + 4 = \frac{1}{2}x + \frac{8}{2}$
$= \frac{1}{2}(x + 8)$

98. $\frac{1}{3}y + 5 = \frac{1}{3}y + \frac{15}{3} = \frac{1}{3}(y + 15)$

99. $\frac{1}{2}x^3 + 2x^2 - 5x = \frac{1}{2}x^3 + \frac{4}{2}x^2 - \frac{10}{2}x$
$= \frac{1}{2}x(x^2 + 4x - 10)$

100. $\frac{1}{3}y^4 - 5y^2 + 2y = \frac{1}{3}y^4 - \frac{15}{3}y^2 + \frac{6}{3}y$
$= \frac{1}{3}y(y^3 - 15y + 6)$

101. $\frac{2}{3}x(x - 3) - 4(x - 3) = \frac{2}{3}x(x - 3) - \frac{12}{3}(x - 3)$
$= \frac{2}{3}(x - 3)(x - 6)$

102. $\frac{4}{5}y(y + 1) - 2(y + 1) = \frac{4}{5}y(y + 1) - \frac{10}{5}(y + 1) = \frac{2}{5}(y + 1)(2y - 5)$

103. $x^2 - 81 = x^2 - 9^2$
$= (x + 9)(x - 9)$

104. $x^2 - 49 = x^2 - 7^2$
$= (x + 7)(x - 7)$

105. $32y^2 - 18 = 2(16y^2 - 9)$
$= 2[(4y)^2 - 3^2]$
$= 2(4y + 3)(4y - 3)$

106. $4 - 36y^2 = 4(1 - 9y^2)$
$= 4[1^2 - (3y)^2]$
$= 4(1 + 3y)(1 - 3y)$

107. $16x^2 - \frac{1}{9} = (4x)^2 - \left(\frac{1}{3}\right)^2$
$= \left(4x + \frac{1}{3}\right)\left(4x - \frac{1}{3}\right)$

108. $\frac{4}{25}y^2 - 64 = \left(\frac{2}{5}y\right)^2 - 8^2$
$= \left(\frac{2}{5}y + 8\right)\left(\frac{2}{5}y - 8\right)$

109. $(x - 1)^2 - 4 = (x - 1)^2 - (2)^2$
$= [(x - 1) + 2][(x - 1) - 2]$
$= (x + 1)(x - 3)$

110. $25 - (z + 5)^2 = 5^2 - (z + 5)^2$
$= (5 - (z + 5))(5 + (z + 5))$
$= (5 - z - 5)(5 + z + 5)$
$= -z(z + 10)$

111. $9u^2 - 4v^2 = (3u)^2 - (2v)^2$
$= (3u + 2v)(3u - 2v)$

112. $25x^2 - 16y^2 = (5x)^2 - (4y)^2$
$= (5x + 4y)(5x - 4y)$

113. $x^2 - 4x + 4 = x^2 - 2(2)x + 2^2$
$= (x - 2)^2$

114. $x^2 + 10x + 25 = x^2 + 2(5)(x) + 5^2 = (x + 5)^2$

115. $4t^2 + 4t + 1 = (2t)^2 + 2(2t)(1) + 1^2$
$= (2t + 1)^2$

116. $9x^2 - 12x + 4 = (3x)^2 - 2(3x)(2) + 2^2 = (3x - 2)^2$

117. $25y^2 - 10y + 1 = (5y)^2 - 2(5y)(1) + 1^2$
$= (5y - 1)^2$

118. $36y^2 - 108y + 81 = 9(4y^2 - 12y + 9)$
$= 9[(2y)^2 - 2(2y)(3) + (3)^2]$
$= 9(2y - 3)^2$

119. $9u^2 + 24uv + 16v^2 = (3u)^2 + 2(3u)(4v) + (4v)^2$
$= (3u + 4v)^2$

120. $4x^2 - 4xy + y^2 = (2x)^2 - 2(2x)y + y^2$
$= (2x - y)^2$

121. $x^2 - \frac{4}{3}x + \frac{4}{9} = x^2 - 2(x)\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2$
$= \left(x - \frac{2}{3}\right)^2$

122. $z^2 + z + \frac{1}{4} = z^2 + 2(z)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$
$= \left(z + \frac{1}{2}\right)^2$

123. $x^3 - 8 = x^3 - 2^3$
$= (x - 2)(x^2 + 2x + 4)$

124. $x^3 - 27 = x^3 - 3^3 = (x - 3)(x^2 + 3x + 9)$

125. $y^3 + 64 = y^3 + 4^3 = (y + 4)(y^2 - 4y + 16)$

126. $z^3 + 125 = z^3 + 5^3 = (z + 5)(z^2 - 5z + 25)$

127. $8t^3 - 1 = (2t)^3 - 1^3$
$= (2t - 1)(4t^2 + 2t + 1)$

128. $27x^3 + 8 = (3x)^3 + 2^3 = (3x + 2)(9x^2 - 6x + 4)$

129. $u^3 + 27v^3 = u^3 + (3v)^3$
$= (u + 3v)(u^2 - 3uv + 9v^2)$

130. $64x^3 - y^3 = (4x)^3 - y^3 = (4x - y)(16x^2 + 4xy + y^2)$

131. $x^2 + x - 2 = (x + 2)(x - 1)$

132. $x^2 + 5x + 6 = (x + 2)(x + 3)$

133. $s^2 - 5s + 6 = (s - 3)(s - 2)$

134. $t^2 - t - 6 = (t + 2)(t - 3)$

135. $20 - y - y^2 = -(y^2 + y - 20)$
$= -(y + 5)(y - 4)$

136. $24 + 5z - z^2 = -(z^2 - 5z - 24) = -(z - 8)(z + 3)$

137. $x^2 - 30x + 200 = (x - 20)(x - 10)$

138. $x^2 - 13x + 42 = (x - 6)(x - 7)$

139. $3x^2 - 5x + 2 = (3x - 2)(x - 1)$

140. $2x^2 - x - 1 = (2x + 1)(x - 1)$

141. $5x^2 + 26x + 5 = (5x + 1)(x + 5)$

142. $12x^2 + 7x + 1 = (3x + 1)(4x + 1)$

143. $-9z^2 + 3z + 2 = -(9z^2 - 3z - 2)$
$= -(3z - 2)(3z + 1)$

144. $-5u^2 - 13u + 6 = -(5u^2 + 13u - 6)$
$= -(5u - 2)(u + 3)$

145. $x^3 - x^2 + 2x - 2 = x^2(x - 1) + 2(x - 1)$
$= (x - 1)(x^2 + 2)$

146. $x^3 + 5x^2 - 5x - 25 = x^2(x + 5) - 5(x + 5)$
$= (x + 5)(x^2 - 5)$

147. $2x^3 - x^2 - 6x + 3 = x^2(2x - 1) - 3(2x - 1)$
$= (2x - 1)(x^2 - 3)$

148. $5x^3 - 10x^2 + 3x - 6 = 5x^2(x - 2) + 3(x - 2)$
$= (x - 2)(5x^2 + 3)$

149. $6 + 2x - 3x^3 - x^4 = 2(3 + x) - x^3(3 + x)$
$= (3 + x)(2 - x^3)$

150. $x^5 + 2x^3 + x^2 + 2 = x^3(x^2 + 2) + (x^2 + 2)$
$= (x^2 + 2)(x^3 + 1)$
$= (x^2 + 2)(x + 1)(x^2 - x + 1)$

151. $6x^3 - 2x + 3x^2 - 1 = 2x(3x^2 - 1) + 1(3x^2 - 1)$
$= (3x^2 - 1)(2x + 1)$

152. $8x^5 - 6x^2 + 12x^3 - 9 = 2x^2(4x^3 - 3) + 3(4x^3 - 3)$
$= (4x^3 - 3)(2x^2 + 3)$

153. $a \cdot c = (3)(8) = 24$. Rewrite the middle term, $10x = 6x + 4x$, since $(6)(4) = 24$ and $6 + 4 = 10$.

$3x^2 + 10x + 8 = 3x^2 + 6x + 4x + 8$
$= 3x(x + 2) + 4(x + 2)$
$= (x + 2)(3x + 4)$

154. $a \cdot c = (2)(9) = 18$. Rewrite the middle term, $9x = 6x + 3x$, since $(6)(3) = 18$ and $6 + 3 = 9$.

$2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$
$= 2x(x + 3) + 3(x + 3)$
$= (x + 3)(2x + 3)$

155. $a \cdot c = (6)(-2) = -12$. Rewrite the middle term, $x = 4x - 3x$, since $4(-3) = -12$ and $4 + (-3) = 1$.

$6x^2 + x - 2 = 6x^2 + 4x - 3x - 2$
$= 2x(3x + 2) - 1(3x + 2)$
$= (2x - 1)(3x + 2)$

156. $a \cdot c = (6)(-15) = -90$. Rewrite the middle term, $-x = -10x + 9x$, since $(-10)(9) = -90$ and $-10 + 9 = -1$.

$6x^2 - x - 15 = 6x^2 - 10x + 9x - 15$
$= 2x(3x - 5) + 3(3x - 5)$
$= (2x + 3)(3x - 5)$

157. $a \cdot c = (15)(2) = 30$. Rewrite the middle term, $-11x = -6x - 5x$, since $(-6)(-5) = 30$ and $(-6) + (-5) = -11$.

$15x^2 - 11x + 2 = 15x^2 - 6x - 5x + 2$
$= 3x(5x - 2) - 1(5x - 2)$
$= (3x - 1)(5x - 2)$

158. $a \cdot c = (12)(1) = 12$. Rewrite the middle term, $-13x = -12x - x$, since $(-12)(-1) = 12$ and $-12 - 1 = -13$.

$12x^2 - 13x + 1 = 12x^2 - 12x - x + 1$
$= 12x(x - 1) - 1(x - 1) = (x - 1)(12x - 1)$

159. $6x^2 - 54 = 6(x^2 - 9)$
$= 6(x + 3)(x - 3)$

160. $12x^2 - 48 = 12(x^2 - 4)$
$= 12(x + 2)(x - 2)$

161. $x^3 - 4x^2 = x^2(x - 4)$

162. $x^3 - 9x = x(x^2 - 9)$
$= x(x + 3)(x - 3)$

163. $x^2 - 2x + 1 = (x - 1)^2$

164. $16 + 6x - x^2 = 16 + 8x - 2x - x^2$
$= (8 - x)(2 + x)$

165. $1 - 4x + 4x^2 = (1 - 2x)^2$

166. $-9x^2 + 6x - 1 = -9x^2 + 3x + 3x - 1$
$= (3x - 1)(-3x + 1)$

167. $2x^2 + 4x - 2x^3 = -2x(-x - 2 + x^2)$
$= -2x(x^2 - x - 2)$
$= -2x(x + 1)(x - 2)$

168. $2y^3 - 7y^2 - 15y = y(2y^2 - 7y - 15)$
$= y(2y^2 - 10y + 3y - 15)$
$= y(2y + 3)(y - 5)$

169. $9x^2 + 10x + 1 = (9x + 1)(x + 1)$

170. $13x + 6 + 5x^2 = 5x^2 + 13x + 6$
$= 5x^2 + 10x + 3x + 6$
$= (5x + 3)(x + 2)$

171. $\frac{1}{81}x^2 + \frac{2}{9}x - 8 = \frac{1}{81}x^2 + \frac{18}{81}x - \frac{648}{81}$
$= \frac{1}{81}(x^2 + 18x - 648)$
$= \frac{1}{81}(x + 36)(x - 18)$

172. $\frac{1}{8}x^2 - \frac{1}{96}x - \frac{1}{16} = \frac{1}{96}\left(12x^2 - x - 6\right)$
$= \frac{1}{96}(4x - 3)(3x + 2)$

173. $3x^3 + x^2 + 15x + 5 = x^2(3x + 1) + 5(3x + 1)$
$= (3x + 1)(x^2 + 5)$

174. $5 - x + 5x^2 - x^3 = 1(5 - x) + x^2(5 - x)$
$= (5 - x)(1 + x^2)$

175. $x^4 - 4x^3 + x^2 - 4x = x(x^3 - 4x^2 + x - 4)$
$= x[x^2(x - 4) + (x - 4)]$
$= x(x - 4)(x^2 + 1)$

176. $3u - 2u^2 + 6 - u^3 = -u^3 - 2u^2 + 3u + 6$
$= -u^2(u + 2) + 3(u + 2)$
$= (u + 2)(-u^2 + 3)$
$= (u + 2)(3 - u^2)$

177. $\frac{1}{4}x^3 + 3x^2 + \frac{3}{4}x + 9 = \frac{1}{4}x^3 + \frac{12}{4}x^2 + \frac{3}{4}x + \frac{36}{4}$
$= \frac{1}{4}(x^3 + 12x^2 + 3x + 36)$
$= \frac{1}{4}[x^2(x + 12) + 3(x + 12)]$
$= \frac{1}{4}(x + 12)(x^2 + 3)$

178. $\frac{1}{5}x^3 + x^2 - x - 5 = \frac{1}{5}(x^3 + 5x^2 - 5x - 25)$
$= \frac{1}{5}[x^2(x + 5) - 5(x + 5)]$
$= \frac{1}{5}(x^2 - 5)(x + 5)$

179. $(t - 1)^2 - 49 = (t - 1)^2 - (7)^2$
$= [(t - 1) + 7][(t - 1) - 7]$
$= (t + 6)(t - 8)$

180. $(x^2 + 1)^2 - 4x^2 = [(x^2 + 1) + 2x][(x^2 + 1) - 2x]$
$= (x^2 + 2x + 1)(x^2 - 2x + 1)$
$= (x + 1)^2(x - 1)^2$

181. $(x^2 + 8)^2 - 36x^2 = (x^2 + 8)^2 - (6x)^2$
$= [(x^2 + 8) - 6x][(x^2 + 8) + 6x]$
$= (x^2 - 6x + 8)(x^2 + 6x + 8)$
$= (x - 4)(x - 2)(x + 4)(x + 2)$

182. $2t^3 - 16 = 2(t^3 - 8) = 2(t - 2)(t^2 + 2t + 4)$

183. $5x^3 + 40 = 5(x^3 + 8)$
$= 5(x^3 + 2^3)$
$= 5(x + 2)(x^2 - 2x + 4)$

184. $4x(2x - 1) + (2x - 1)^2 = (2x - 1)[4x + (2x - 1)]$
$= (2x - 1)(6x - 1)$

185. $5(3 - 4x)^2 - 8(3 - 4x)(5x - 1) = (3 - 4x)[5(3 - 4x) - 8(5x - 1)]$
$= (3 - 4x)[15 - 20x - 40x + 8]$
$= (3 - 4x)(23 - 60x)$

186. $2(x + 1)(x - 3)^2 - 3(x + 1)^2(x - 3) = (x + 1)(x - 3)[2(x - 3) - 3(x + 1)]$
$= (x + 1)(x - 3)[2x - 6 - 3x - 3]$
$= (x + 1)(x - 3)(-x - 9)$
$= -(x + 1)(x - 3)(x + 9)$

187. $7(3x + 2)^2(1 - x)^2 + (3x + 2)(1 - x)^3 = (3x + 2)(1 - x)^2[7(3x + 2) + (1 - x)]$
$= (3x + 2)(1 - x)^2(21x + 14 + 1 - x)$
$= (3x + 2)(1 - x)^2(20x + 15)$
$= 5(3x + 2)(1 - x)^2(4x + 3)$

188. $7x(2)(x^2 + 1)(2x) - (x^2 + 1)^2(7) = 7(x^2 + 1)[4x^2 - (x^2 + 1)]$
$= 7(x^2 + 1)(3x^2 - 1)$

189. $3(x - 2)^2(x + 1)^4 + (x - 2)^3(4)(x + 1)^3 = (x - 2)^2(x + 1)^3[3(x + 1) + 4(x - 2)]$
$= (x - 2)^2(x + 1)^3(3x + 3 + 4x - 8)$
$= (x - 2)^2(x + 1)^3(7x - 5)$

190. $2x(x - 5)^4 - x^2(4)(x - 5)^3 = 2x(x - 5)^3[(x - 5) - 2x]$
$= 2x(x - 5)^3(-x - 5)$
$= -2x(x - 5)^3(x + 5)$

191. $5(x^6 + 1)^4(6x^5)(3x + 2)^3 + 3(3x + 2)^2(3)(x^6 + 1)^5 = 3(x^6 + 1)^4(3x + 2)^2[10x^5(3x + 2) + 3(x^6 + 1)]$
$= 3(x^6 + 1)^4(3x + 2)^2(30x^6 + 20x^5 + 3x^6 + 3)$
$= 3(x^6 + 1)^4(3x + 2)^2(33x^6 + 20x^5 + 3)$
$= 3[(x^2)^3 + 1]^4(3x + 2)^2(33x^6 + 20x^5 + 3)$
$= 3[(x^2 + 1)(x^4 - x^2 + 1)]^4(3x + 2)^2(33x^6 + 20x^5 + 3)$
$= 3(x^2 + 1)^4(x^4 - x^2 + 1)^4(3x + 2)^2(33x^6 + 20x^5 + 3)$

192. $\frac{x^2}{2}(x^2 + 1)^4 - (x^2 + 1)^5 = (x^2 + 1)^4\left[\frac{x^2}{2} - (x^2 + 1)\right]$
$= (x^2 + 1)^4\left(-\frac{x^2}{2} - 1\right)$
$= -(x^2 + 1)^4\left(\frac{x^2}{2} + 1\right)$

193. For $x^2 + bx - 15$ to be factorable, b must equal $m + n$ where $mn = -15$.

Factors of -15	Sum of factors
$(15)(-1)$	$15 + (-1) = 14$
$(-15)(1)$	$-15 + 1 = -14$
$(3)(-5)$	$3 + (-5) = -2$
$(-3)(5)$	$-3 + 5 = 2$

The possible b-values are $14, -14, -2$, or 2.

194. For $x^2 + bx + 50$ to be factorable, b must equal $m + n$ where $mn = 50$.

Factors of 50	Sum of factors
$(1)(50)$	$1 + 50 = 51$
$(-1)(-50)$	$-1 + (-50) = -51$
$(5)(10)$	$5 + 10 = 15$
$(-5)(-10)$	$-5 + (-10) = -15$
$(2)(25)$	$2 + 25 = 27$
$(-2)(-25)$	$-2 + (-25) = -27$

The possible b-values are $-51, 51, -15, 15, -27, 27$.

195. For $x^2 + bx - 12$ to be factorable, b must equal $m + n$ where $mn = -12$.

Factors of -12	Sum of factors
$(12)(-1)$	$12 + (-1) = 11$
$(-12)(1)$	$-12 + 1 = -11$
$(2)(-6)$	$2 + (-6) = -4$
$(-2)(6)$	$-2 + 6 = 4$
$(3)(-4)$	$3 + (-4) = -1$
$(-3)(4)$	$-3 + 4 = 1$

The possible b-values are $11, -11, -4, 4, -1, 1$.

196. For $x^2 + bx + 24$ to be factorable, b must equal $m + n$ where $mn = 24$.

Factors of 24	Sum of factors
$(1)(24)$	$1 + 24 = 25$
$(-1)(-24)$	$-1 + (-24) = -25$
$(2)(12)$	$2 + 12 = 14$
$(-2)(-12)$	$-2 + (-12) = -14$
$(3)(8)$	$3 + 8 = 11$
$(-3)(-8)$	$-3 + (-8) = -11$
$(4)(6)$	$4 + 6 = 10$
$(-4)(-6)$	$-4 + (-6) = -10$

The possible b-values are $25, -25, 14, -14$ $11, -11, 10, -10$.

197. For $2x^2 + 5x + c$ to be factorable, the factors of $2c$ must add up to 5.

Possible c-values	$2c$	Factors of $2c$ that add up to 5
2	4	$(1)(4) = 4$ and $1 + 4 = 5$
3	6	$(2)(3) = 6$ and $2 + 3 = 5$
-3	-6	$(6)(-1) = -6$ and $6 + (-1) = 5$
-7	-14	$(7)(-2) = -14$ and $7 + (-2) = 5$
-12	-24	$(8)(-3) = -24$ and $8 + (-3) = 5$

These are a few possible c-values. There are *many* correct answers.

If $c = 2$: $2x^2 + 5x + 2 = (2x + 1)(x + 2)$

If $c = 3$: $2x^2 + 5x + 3 = (2x + 3)(x + 1)$

If $c = -3$: $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

If $c = -7$: $2x^2 + 5x - 7 = (2x + 7)(x - 1)$

If $c = -12$: $2x^2 + 5x - 12 = (2x - 3)(x + 4)$

198. For $3x^2 - 10x + c$ to be factorable, the factors of $3c$ must add up to -10.

Possible c-values	$3c$	Factors of $3c$ that add up to -10
3	9	$(-1)(-9) = 9$ and $-1 + (-9) = -10$
-8	-24	$(-12)(2) = -24$ and $-12 + 2 = -10$
8	24	$(-6)(-4) = 24$ and $-6 + (-4) = -10$

These are a few possible c-values. There are *many* correct answers.

If $c = 3$: $3x^2 - 10x + 3 = (3x - 1)(x - 3)$

If $c = -8$: $3x^2 - 10x - 8 = (3x + 2)(x - 4)$

If $c = 8$: $3x^2 - 10x + 8 = (3x - 4)(x - 2)$

199. For $3x^2 - x + c$ to be factorable, the factors of $3c$ must add up to -1.

Possible c-values	$3c$	Factors of $3c$ must add up to -1
-2	-6	$(2)(-3) = -6$ and $2 + (-3) = -1$
-4	-12	$(3)(-4) = -12$ and $3 + (-4) = -1$
-10	-30	$(5)(-6) = -30$ and $5 + (-6) = -1$

These are a few possible c-values. There are *many* correct answers.

If $c = -2$: $3x^2 - x - 2 = (3x + 2)(x - 1)$

If $c = -4$: $3x^2 - x - 4 = (3x - 4)(x + 1)$

If $c = -10$: $3x^2 - x - 10 = (3x + 5)(x - 2)$

200. For $2x^2 + 9x + c$ to be factorable, the factors of $2c$ must add up to 9. There are many possibilities.

Possible c-values	$2c$	Factors of $2c$ that add up to 9
4	8	$(1)(8) = 8$ and $1 + 8 = 9$
7	14	$(2)(7) = 14$ and $2 + 7 = 9$
9	18	$(3)(6) = 18$ and $3 + 6 = 9$
10	20	$(4)(5) = 20$ and $4 + 5 = 9$
-11	-22	$(-2)(11) = -22$ and $-2 + 11 = 9$
-18	-36	$(-3)(12) = -36$ and $-3 + 12 = 9$

These are a few possible c-values.

$2x^2 + 9x + 4 = (2x + 1)(x + 4)$

$2x^2 + 9x + 7 = (2x + 7)(x + 1)$

$2x^2 + 9x + 9 = (2x + 3)(x + 3)$

$2x^2 + 9x + 10 = (2x + 5)(x + 2)$

$2x^2 + 9x - 11 = (2x + 11)(x - 1)$

$2x^2 + 9x - 18 = (2x - 3)(x + 6)$

201. (a) Profit = Revenue − Cost

$$\text{Profit} = 95x - (73x + 25{,}000)$$
$$= 95x - 73x - 25{,}000 = 22x - 25{,}000$$

(b) For $x = 5000$:

$$\text{Profit} = 22(5000) - 25{,}000$$
$$= 110{,}000 - 25{,}000 = \$85{,}000$$

202. (a) Profit = Revenue − Cost

$$P = 36x - (460 + 12x)$$
$$= 36x - 460 - 12x$$
$$= 24x - 460$$

(b) When $x = 42$, $P = 24(42) - 460 = \$548$.

203. (a) $500(1 + r)^2 = 500(r + 1)^2 = 500(r^2 + 2r + 1)$

$$= 500r^2 + 1000r + 500$$

(b)

r	$2\frac{1}{2}\%$	3%	4%	$4\frac{1}{2}\%$	5%
$500(1 + r)^2$	\$525.31	\$530.45	\$540.80	\$546.01	\$551.25

(c) As r increases, the amount increases.

204. (a) $1200(1 + r)^3 = 1200(1 + 3r + 3r^2 + r^3)$

$$= 1200(r^3 + 3r^2 + 3r + 1)$$
$$= 1200r^3 + 3600r^2 + 3600r + 1200$$

(b)

r	2%	3%	$3\frac{1}{2}\%$	4%	$4\frac{1}{2}\%$
$1200(1 + r)^3$	\$1273.45	\$1311.27	\$1330.46	\$1349.84	\$1369.40

(c) Amount increases with increasing r.

205. (a) $V = l \cdot w \cdot h = (26 - 2x)(18 - 2x)(x)$

$$= 2(13 - x)(2)(9 - x)(x)$$
$$= 4x(-1)(x - 13)(-1)(x - 9)$$
$$= 4x(x - 13)(x - 9)$$
$$= 4x^3 - 88x^2 + 468x$$

(b)

x (cm)	1	2	3
V (cm^3)	384	616	720

206. (a) Volume = length × width × height

$$= \tfrac{1}{2}(45 - 3x)(15 - 2x)x$$
$$= \tfrac{1}{2}(45 - 3x)(15x - 2x^2)$$
$$= \tfrac{1}{2}[675x - 90x^2 - 45x^2 + 6x^3]$$
$$= \tfrac{1}{2}(6x^3 - 135x^2 + 675x)$$

x (cm)	3	5	7
Volume (cm^3)	486	375	84

(b) When $x = 3$: $V = \frac{1}{2}[6(3)^3 - 135(3)^2 + 675(3)] = \frac{1}{2}[6 \cdot 27 - 135 \cdot 9 + 2025]$

$$= \tfrac{1}{2}[162 - 1215 + 2025] = \tfrac{1}{2}(972)$$
$$= 486 \text{ cubic centimeters}$$

When $x = 5$: $V = \frac{1}{2}[6(5)^3 - 135(5)^2 + 675(5)] = \frac{1}{2}[6 \cdot 125 - 135 \cdot 25 + 3375]$

$$= \tfrac{1}{2}[750 - 3375 + 3375] = \tfrac{1}{2}(750)$$
$$= 375 \text{ cubic centimeters}$$

When $x = 7$: $V = \frac{1}{2}[6(7)^3 - 135(7)^2 + 675(7)] = \frac{1}{2}[6 \cdot 343 - 135 \cdot 49 + 4725]$

$$= \tfrac{1}{2}[2058 - 6615 + 4725] = \tfrac{1}{2}(168)$$
$$= 84 \text{ cubic centimeters}$$

207. Area = length × width

$= (2x + 14)(22)$

$= (2x)(22) + (14)(22)$

$= 44x + 308$

208. $A = (18 + 2x)(14 + x) = 252 + 18x + 28x + 2x^2$

$= 2x^2 + 46x + 252$

209. (a) Area of shaded region = Area of outer rectangle − Area of inner rectangle

$A = 2x(2x + 6) - x(x + 4)$

$= 4x^2 + 12x - x^2 - 4x$

$= 3x^2 + 8x$

(b) Area of shaded region = Area of outer triangle − Area of inner triangle

$A = \frac{1}{2}(9x)(12x) - \frac{1}{2}(6x)(8x)$

$= 54x^2 - 24x^2$

$= 30x^2$

210. (a) $T = R + B = 1.1x + (0.0475x^2 - 0.001x + 0.23)$

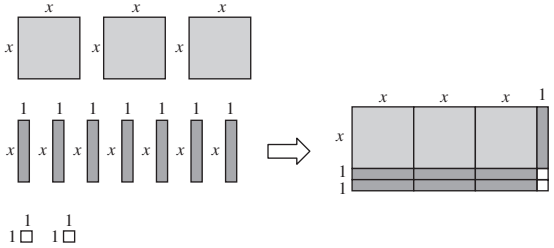
$= 0.0475x^2 + 1.099x + 0.23$

(c) Stopping distance increases at an accelerating rate as speed increases.

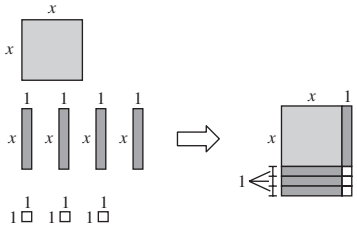
(b)

x mi/hr	30	40	55
T feet	75.95	120.19	204.36

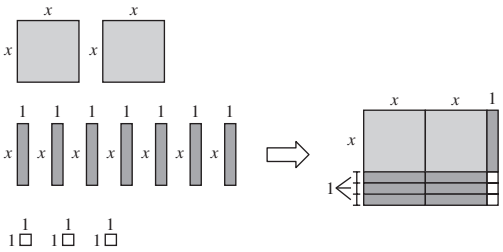
211. $3x^2 + 7x + 2 = (3x + 1)(x + 2)$



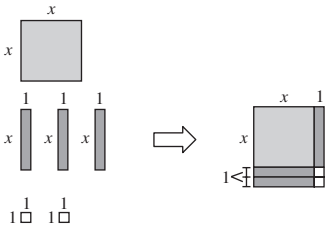
212. $x^2 + 4x + 3 = (x + 3)(x + 1)$



213. $2x^2 + 7x + 3 = (2x + 1)(x + 3)$



214. $x^2 + 3x + 2 = (x + 2)(x + 1)$



215. $A = \pi(r + 2)^2 - \pi r^2$

$= \pi[(r + 2)^2 - r^2]$

$= \pi[r^2 + 4r + 4 - r^2]$

$= \pi(4r + 4)$

$= 4\pi(r + 1)$

216. Area $= (2r)^2 - \pi r^2$

$= 4r^2 - \pi r^2$

$= r^2(4 - \pi)$

217. $A = 8(18) - 4x^2$

$= 4(36 - x^2)$

$= 4(6 - x)(6 + x)$

218. Area $= \frac{1}{2}(x+3)\left(\frac{5}{4}\right)(x+3) - \frac{1}{2}(5)(4)$

$= \frac{5}{8}(x^2 + 6x + 9) - \frac{5}{8}(16)$

$= \frac{5}{8}(x^2 + 6x + 9 - 16)$

$= \frac{5}{8}(x^2 + 6x - 7)$

$= \frac{5}{8}(x+7)(x-1)$

219. (a) $V = \pi R^2 h - \pi r^2 h$

$= \pi h(R^2 - r^2)$

$= \pi h(R - r)(R + r)$

(b) The average radius is $(R + r)/2$. The thickness of the tank is $R - r$.

$$V = \pi h(R-r)(R+r) = 2\pi\left(\frac{R+r}{2}\right)(R-r)h$$

$= 2\pi$(average radius)(thickness)h

220. $kQx - kx^2 = kx(Q - x)$

221. False. $(4x^2 + 1)(3x + 1) = 12x^3 + 4x^2 + 3x + 1$

222. False.

$(4x + 3) + (-4x + 6) = 4x + 3 - 4x + 6$

$= 3 + 6 = 9$

223. True. $a^2 - b^2 = (a + b)(a - b)$

224. False. A perfect square trinomial can be factored as the binomial sum squared.

225. Since $x^m x^n = x^{m+n}$, the degree of the product is $m + n$.

226. If the degree of one polynomial is m and the degree of the second polynomial is n (and $n > m$), the degree of the sum of the polynomials is n.

227. The unknown polynomial may be found by adding $-x^3 + 3x^2 + 2x - 1$ and $5x^2 + 8$:

$(-x^3 + 3x^2 + 2x - 1) + (5x^2 + 8) = -x^3 + (3x^2 + 5x^2) + 2x + (-1 + 8)$

$= -x^3 + 8x^2 + 2x + 7$

228. $(x + y)^2 \neq x^2 + y^2$

Let $x = 3$ and $y = 4$.

$(3 + 4)^2 = (7)^2 = 49$

$3^2 + 4^2 = 9 + 16 = 25$ $>$ Not Equal

If either x or y is zero, then $(x + y)^2$ would equal $x^2 + y^2$.

229. $x^{2n} - y^{2n} = (x^n)^2 - (y^n)^2$

$= (x^n + y^n)(x^n - y^n)$

This is not completely factored unless $n = 1$.

For $n = 2$: $(x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x + y)(x - y)$

For $n = 3$: $(x^3 + y^3)(x^3 - y^3) = (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$

For $n = 4$: $(x^4 + y^4)(x^4 - y^4) = (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

230. $x^{3n} + y^{3n} = (x^n)^3 + (y^n)^3 = (x^n + y^n)(x^{2n} - x^n y^n + y^{2n})$

Depending on the value of n, this may factor further.

231. $x^{3n} - y^{2n} = (x^n)^3 - (y^n)^2 = x^{3n} - y^{2n}$ is completely factored. For integer values of n greater than 4, the factorizations become more complicated.

232. Answers will vary. A possible answer: A polynomial is in factored form when written as a product of polynomials of lesser degree than the given polynomial.

233. Answers will vary. Some examples:

$x^2 - 3;\ x^2 + x + 1;\ x^2 + 16$

Appendix A.4 Rational Expressions

- You should be able to find the domain of a rational expression.
- You should know that a rational expression is the quotient of two polynomials.
- You should be able to simplify rational expressions by reducing them to lowest terms. This may involve factoring both the numerator and the denominator.
- You should be able to add, subtract, multiply, and divide rational expressions.
- You should be able to simplify complex fractions.
- You should be able to simplify expressions with negative or fraction exponents.

Vocabulary Check

1. domain **2.** rational expression **3.** complex

4. smaller **5.** equivalent **6.** difference quotient

1. The domain of the polynomial $3x^2 - 4x + 7$ is the set of all real numbers.

2. The domain of the polynomial $2x^2 + 5x - 2$ is the set of all real numbers.

3. The domain of the polynomial $4x^3 + 3$, $x \geq 0$ is the set of non-negative real numbers, since the polynomial is restricted to that set.

4. The domain of the polynomial $6x^2 - 9$, $x > 0$ is the set of all positive real numbers because the polynomial is restricted to that set.

5. The domain of $1/(x - 2)$ is the set of all real numbers x such that $x \neq 2$.

6. The domain of $(x + 1)/(2x + 1)$ is the set of all real numbers such that $x \neq -1/2$.

7. The domain of $\sqrt{x + 1}$ is the set of all real numbers x such that $x \geq -1$.

8. The domain of $\sqrt{6 - x}$ is the set of all real numbers x such that $x \leq 6$.

9. $\dfrac{5}{2x} = \dfrac{5(3x)}{(2x)(3x)} = \dfrac{5(3x)}{6x^2}, \quad x \neq 0$

The missing factor is $3x$, $x \neq 0$.

10. $\dfrac{3}{4} = \dfrac{3(x + 1)}{4(x + 1)}$

The missing factor is $(x + 1)$, where $x \neq -1$.

11. $\dfrac{15x^2}{10x} = \dfrac{5x(3x)}{5x(2)} = \dfrac{3x}{2}, \quad x \neq 0$

12. $\dfrac{18y^2}{60y^5} = \dfrac{6y^2(3)}{6y^2(10y^3)} = \dfrac{3}{10y^3}$

13. $\dfrac{3xy}{xy + x} = \dfrac{x(3y)}{x(y + 1)} = \dfrac{3y}{y + 1}, \quad x \neq 0$

14. $\dfrac{2x^2y}{xy - y} = \dfrac{2x^2y}{y(x - 1)} = \dfrac{2x^2}{x - 1}$

15. $\dfrac{4y - 8y^2}{10y - 5} = \dfrac{-4y(2y - 1)}{5(2y - 1)} = -\dfrac{4y}{5}, \quad y \neq \dfrac{1}{2}$

16. $\dfrac{9x^2 + 9x}{2x + 2} = \dfrac{9x(x + 1)}{2(x + 1)} = \dfrac{9x}{2}, \quad x \neq -1$

17. $\dfrac{x - 5}{10 - 2x} = \dfrac{x - 5}{-2(x - 5)} = -\dfrac{1}{2}, \quad x \neq 5$

18. $\dfrac{12 - 4x}{x - 3} = \dfrac{4(3 - x)}{x - 3} = -4, \quad x \neq 3$

19. $\dfrac{y^2 - 16}{y + 4} = \dfrac{(y + 4)(y - 4)}{y + 4} = y - 4, \quad y \neq -4$

20. $\dfrac{x^2 - 25}{5 - x} = \dfrac{(x + 5)(x - 5)}{-1(x - 5)} = -(x + 5), \quad x \neq 5$

21. $\dfrac{x^3 + 5x^2 + 6x}{x^2 - 4} = \dfrac{x(x+2)(x+3)}{(x+2)(x-2)} = \dfrac{x(x+3)}{x-2}, \quad x \neq -2$

22. $\dfrac{x^2 + 8x - 20}{x^2 + 11x + 10} = \dfrac{(x+10)(x-2)}{(x+10)(x+1)} = \dfrac{x-2}{x-1}, \quad x \neq -10$

23. $\dfrac{y^2 - 7y + 12}{y^2 + 3y - 18} = \dfrac{(y-3)(y-4)}{(y+6)(y-3)} = \dfrac{y-4}{y+6}, \quad y \neq 3$

24. $\dfrac{x^2 - 7x + 6}{x^2 + 11x + 10} = \dfrac{(x-6)(x-1)}{(x+10)(x+1)}$

25.
$$\begin{aligned}\frac{2 - x + 2x^2 - x^3}{x^2 - 4} &= \frac{(2-x) + x^2(2-x)}{(x+2)(x-2)} \\ &= \frac{(2-x)(1+x^2)}{(x+2)(x-2)} \\ &= \frac{-(x-2)(x^2+1)}{(x+2)(x-2)} \\ &= -\frac{x^2+1}{x+2}, \quad x \neq 2\end{aligned}$$

26.
$$\begin{aligned}\frac{x^2 - 9}{x^3 + x^2 - 9x - 9} &= \frac{x^2 - 9}{x^2(x+1) - 9(x+1)} \\ &= \frac{x^2 - 9}{(x^2 - 9)(x+1)} \\ &= \frac{1}{x+1}, \quad x \neq \pm 3\end{aligned}$$

27. $\dfrac{z^3 - 8}{z^2 + 2z + 4} = \dfrac{(z-2)(z^2 + 2z + 4)}{z^2 + 2z + 4} = z - 2$

28.
$$\begin{aligned}\frac{y^3 - 2y^2 - 3y}{y^3 + 1} &= \frac{y(y-3)(y+1)}{(y+1)(y^2 - y + 1)} \\ &= \frac{y(y-3)}{y^2 - y + 1}, \quad y \neq -1\end{aligned}$$

29.

x	0	1	2	3	4	5	6
$\dfrac{x^2 - 2x - 3}{x - 3}$	1	2	3	Undef.	5	6	7
$x + 1$	1	2	3	4	5	6	7

The expressions are equivalent except at $x = 3$.

30.

x	0	1	2	3	4	5	6
$\dfrac{x-3}{x^2 - x - 6}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	Undef.	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$
$\dfrac{1}{x+2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$

The expressions are equivalent except at $x = 3$.

31. $\dfrac{5x^3}{2x^3 + 4} = \dfrac{5x^3}{2(x^3 + 2)}$

There are no common factors so this expression cannot be simplified. In this case factors of terms were incorrectly cancelled.

32. $\dfrac{x^3 + 25x}{x^2 - 2x - 15} = \dfrac{x(x^2 + 25)}{(x-5)(x+3)}$

The expression cannot be simplified.

33. $\dfrac{\pi r^2}{(2r)^2} = \dfrac{\pi r^2}{4r^2} = \dfrac{\pi}{4}, \quad r \neq 0$

34. Area of shaded portion: $\left(\dfrac{x+5}{2}\right)^2 = \dfrac{(x+5)^2}{4}$

Area of total figure: $(2x + 3)(x + 5)$

Ratio: $\dfrac{\frac{(x+5)^2}{4}}{(2x+3)(x+5)} = \dfrac{\frac{(x+5)}{4}}{(2x+3)} = \dfrac{x+5}{4(2x+3)}$

35. $\dfrac{5}{x-1} \cdot \dfrac{x-1}{25(x-2)} = \dfrac{1}{5(x-2)}, \quad x \neq 1$

36.
$$\begin{aligned}\frac{x+13}{x^3(3-x)} \cdot \frac{x(x-3)}{5} &= \frac{x+13}{x^3(x-3)(-1)} \cdot \frac{x(x-3)}{5} \\ &= \frac{x+13}{-5x^2} = -\frac{x+13}{5x^2}, \quad x \neq 3\end{aligned}$$

37. $\dfrac{r}{r-1}\cdot\dfrac{r^2-1}{r^2}=\dfrac{r(r+1)(r-1)}{r^2(r-1)}=\dfrac{r+1}{r},\ r\neq 1, r\neq 0$

38. $\dfrac{4y-16}{5y+15}\cdot\dfrac{2y+6}{4-y}=\dfrac{4(y-4)}{5(y+3)}\cdot\dfrac{2(y+3)}{(-1)(y-4)}$

$=\dfrac{8}{-5}=-\dfrac{8}{5},\ y\neq -3, 4$

39. $\dfrac{t^2-t-6}{t^2+6t+9}\cdot\dfrac{t+3}{t^2-4}=\dfrac{(t-3)(t+2)(t+3)}{(t+3)^2(t+2)(t-2)}=\dfrac{t-3}{(t+3)(t-2)},\quad t\neq -2$

40. $\dfrac{x^2+xy-2y^2}{x^3+x^2y}\cdot\dfrac{x}{x^2+3xy+2y^2}=\dfrac{(x+2y)(x-y)}{x^2(x+y)}\cdot\dfrac{x}{(x+2y)(x+y)}=\dfrac{x-y}{x(x+y)^2},\quad x\neq -2y$

41. $\dfrac{x^2-36}{x}\div\dfrac{x^3-6x^2}{x^2+x}=\dfrac{x^2-36}{x}\cdot\dfrac{x^2+x}{x^3-6x^2}$

$=\dfrac{(x+6)(x-6)}{x}\cdot\dfrac{x(x+1)}{x^2(x-6)}$

$=\dfrac{(x+6)(x+1)}{x^2},\quad x\neq 6$

42. $\dfrac{x^2-14x+49}{x^2-49}\div\dfrac{3x-21}{x+7}=\dfrac{(x-7)(x-7)}{(x+7)(x-7)}\cdot\dfrac{x+7}{3(x-7)}$

$=\dfrac{1}{3},\ x\neq\pm 7$

43. $\dfrac{5}{x-1}+\dfrac{x}{x-1}=\dfrac{5+x}{x-1}=\dfrac{x+5}{x-1}$

44. $\dfrac{2x-1}{x+3}+\dfrac{1-x}{x+3}=\dfrac{2x-1+1-x}{x+3}=\dfrac{x}{x+3}$

45. $6-\dfrac{5}{x+3}=\dfrac{6(x+3)}{(x+3)}-\dfrac{5}{x+3}$

$=\dfrac{6(x+3)-5}{x+3}$

$=\dfrac{6x+18-5}{x+3}$

$=\dfrac{6x+13}{x+3}$

46. $\dfrac{3}{x-1}-5=\dfrac{3}{x-1}-\dfrac{5(x-1)}{x-1}$

$=\dfrac{3-5(x-1)}{x-1}$

$=\dfrac{3-5x+5}{x-1}$

$=\dfrac{8-5x}{x-1}$

47. $\dfrac{3}{x-2}+\dfrac{5}{2-x}=\dfrac{3}{x-2}-\dfrac{5}{x-2}=-\dfrac{2}{x-2}$

48. $\dfrac{2x}{x-5}-\dfrac{5}{5-x}=\dfrac{2x}{x-5}-\dfrac{5(-1)}{(-1)(5-x)}$

$=\dfrac{2x}{x-5}-\dfrac{-5}{x-5}=\dfrac{2x+5}{x-5}$

49. $\dfrac{1}{x^2-x-2}-\dfrac{x}{x^2-5x+6}=\dfrac{1}{(x-2)(x+1)}-\dfrac{x}{(x-2)(x-3)}$

$=\dfrac{(x-3)-x(x+1)}{(x+1)(x-2)(x-3)}=\dfrac{x-3-x^2-x}{(x+1)(x-2)(x-3)}$

$=\dfrac{-x^2-3}{(x+1)(x-2)(x-3)}=-\dfrac{x^2+3}{(x+1)(x-2)(x-3)}$

50. $\dfrac{2}{x^2-x-2}+\dfrac{10}{x^2+2x-8}=\dfrac{2}{(x-2)(x+1)}+\dfrac{10}{(x+4)(x-2)}$

$=\dfrac{2(x+4)}{(x-2)(x+1)(x+4)}+\dfrac{10(x+1)}{(x-2)(x+1)(x+4)}$

$=\dfrac{2x+8+10x+10}{(x-2)(x+1)(x+4)}=\dfrac{12x+18}{(x-2)(x+1)(x+4)}=\dfrac{6(2x+3)}{(x-2)(x+1)(x+4)}$

51. $$-\frac{1}{x}+\frac{2}{x^2+1}+\frac{1}{x^3+x}=\frac{-(x^2+1)}{x(x^2+1)}+\frac{2x}{x(x^2+1)}+\frac{1}{x(x^2+1)}$$
$$=\frac{-x^2-1+2x+1}{x(x^2+1)}=\frac{-x^2+2x}{x(x^2+1)}=\frac{-x(x-2)}{x(x^2+1)}$$
$$=-\frac{x-2}{x^2+1}=\frac{2-x}{x^2+1},\quad x\neq 0$$

52. $$\frac{2}{x+1}+\frac{2}{x-1}+\frac{1}{x^2-1}=\frac{2}{x+1}+\frac{2}{x-1}+\frac{1}{(x+1)(x-1)}$$
$$=\frac{2(x-1)}{(x+1)(x-1)}+\frac{2(x+1)}{(x+1)(x-1)}+\frac{1}{(x+1)(x-1)}$$
$$=\frac{2x-2+2x+2+1}{(x+1)(x-1)}=\frac{4x+1}{(x+1)(x-1)}$$

53. $$\frac{x+4}{x+2}-\frac{3x-8}{x+2}=\frac{(x+4)-(3x-8)}{x+2}$$
$$=\frac{x+4-3x+8}{x+2}=\frac{-2x+12}{x+2}=\frac{-2(x-6)}{x+2}$$

The error was incorrect subtraction in the numerator.

54. $$\frac{6-x}{x(x+2)}+\frac{x+2}{x^2}+\frac{8}{x^2(x+2)}=\frac{x(6-x)}{x^2(x+2)}+\frac{(x+2)^2}{x^2(x+2)}+\frac{8}{x^2(x+2)}$$
$$=\frac{6x-x^2+x^2+4x+4+8}{x^2(x+2)}=\frac{10x+12}{x^2(x+2)}=\frac{2(5x+6)}{x^2(x+2)}$$

The error was an incorrect expansion of $(x+2)^2$ in the numerator.

55. $$\frac{\left(\frac{x}{2}-1\right)}{(x-2)}=\frac{\left(\frac{x}{2}-\frac{2}{2}\right)}{\left(\frac{x-2}{1}\right)}$$
$$=\frac{x-2}{2}\cdot\frac{1}{x-2}$$
$$=\frac{1}{2},\quad x\neq 2$$

56. $$\frac{(x-4)}{\left(\frac{x}{4}-\frac{4}{x}\right)}=\frac{\left(\frac{x-4}{1}\right)}{\left(\frac{x^2}{4x}-\frac{16}{4x}\right)}=\frac{\left(\frac{x-4}{1}\right)}{\left(\frac{x^2-16}{4x}\right)}$$
$$=\frac{x-4}{1}\cdot\frac{4x}{x^2-16}$$
$$=\frac{x-4}{1}\cdot\frac{4x}{(x+4)(x-4)}=\frac{4x}{x+4},\ x\neq 0,4$$

57. $$\frac{\left[\frac{x^2}{(x+1)^2}\right]}{\left[\frac{x}{(x+1)^3}\right]}=\frac{x^2}{(x+1)^2}\cdot\frac{(x+1)^3}{x}$$
$$=x(x+1),\ \ x\neq -1,0$$

58. $$\frac{\left(\frac{x^2-1}{x}\right)}{\left[\frac{(x-1)^2}{x}\right]}=\frac{x^2-1}{x}\cdot\frac{x}{(x-1)^2}$$
$$=\frac{(x+1)(x-1)}{x}\cdot\frac{x}{(x-1)(x-1)}$$
$$=\frac{x+1}{x-1},\ x\neq 0$$

59. $$\frac{\left(\sqrt{x}-\frac{1}{2\sqrt{x}}\right)}{\sqrt{x}}=\frac{\left(\sqrt{x}-\frac{1}{2\sqrt{x}}\right)}{\sqrt{x}}\cdot\frac{2\sqrt{x}}{2\sqrt{x}}=\frac{2x-1}{2x},\quad x>0$$

60. $$\frac{\dfrac{t^2}{\sqrt{t^2+1}}-\sqrt{t^2+1}}{t^2}=\frac{\left[\dfrac{t^2}{\sqrt{t^2+1}}-\sqrt{t^2+1}\right]}{t^2}\cdot\frac{\sqrt{t^2+1}}{\sqrt{t^2+1}}$$
$$=\frac{t^2-(t^2+1)}{t^2\sqrt{t^2+1}}=-\frac{1}{t^2\sqrt{t^2+1}}$$

61. $x^5-2x^{-2}=x^{-2}(x^7-2)=\dfrac{x^7-2}{x^2}$

62. $x^5-5x^{-3}=x^{-3}(x^8-5)=\dfrac{x^8-5}{x^3}$

63. $x^2(x^2+1)^{-5}-(x^2+1)^{-4}=(x^2+1)^{-5}\left[x^2-(x^2+1)\right]=-\dfrac{1}{(x^2+1)^5}$

64. $$2x(x-5)^{-3}-4x^2(x-5)^{-4}=-2x(x-5)^{-4}[-(x-5)+2x]$$
$$=-2x(x-5)^{-4}(-x+5+2x)$$
$$=\frac{-2x(x+5)}{(x-5)^4}$$

65. $2x^2(x-1)^{1/2}-5(x-1)^{-1/2}=(x-1)^{-1/2}\left[2x^2(x-1)^1-5\right]=\dfrac{2x^3-2x^2-5}{(x-1)^{1/2}}$

66. $4x^3(2x-1)^{3/2}-2x(2x-1)^{-1/2}=(2x-1)^{-1/2}[4x^3(2x-1)^2-2x]=\dfrac{4x^3(2x-1)^2-2x}{(2x-1)^{1/2}}$

67. $\dfrac{3x^{1/3}-x^{-2/3}}{3x^{-2/3}}=\dfrac{3x^{1/3}-x^{-2/3}}{3x^{-2/3}}\cdot\dfrac{x^{2/3}}{x^{2/3}}=\dfrac{3x^1-x^0}{3x^0}=\dfrac{3x-1}{3},\ x\neq 0$

68. $$\frac{-x^3(1-x^2)^{-1/2}-2x(1-x^2)^{1/2}}{x^4}=\frac{\dfrac{-x^3}{(1-x^2)^{1/2}}-2x(1-x^2)^{1/2}}{x^4}$$
$$=\frac{\dfrac{-x^3}{(1-x^2)^{1/2}}-\dfrac{2x(1-x^2)^{1/2}(1-x^2)^{1/2}}{(1-x^2)^{1/2}}}{x^4}=\frac{\dfrac{-x^3-2x(1-x^2)}{(1-x^2)^{1/2}}}{x^4}$$
$$=\frac{-x^3-2x+2x^3}{(1-x^2)^{1/2}}\cdot\frac{1}{x^4}=\frac{x^3-2x}{(1-x^2)^{1/2}}\cdot\frac{1}{x^4}$$
$$=\frac{x(x^2-2)}{x^4(1-x^2)^{1/2}}=\frac{x^2-2}{x^3(1-x^2)^{1/2}}$$

69. $$\frac{\left(\dfrac{1}{x+h}-\dfrac{1}{x}\right)}{h}=\frac{\left(\dfrac{1}{x+h}-\dfrac{1}{x}\right)}{h}\cdot\frac{x(x+h)}{x(x+h)}$$
$$=\frac{x-(x+h)}{hx(x+h)}$$
$$=\frac{-h}{hx(x+h)}$$
$$=-\frac{1}{x(x+h)},\quad h\neq 0$$

70. $$\frac{\left[\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right]}{h}=\frac{\left[\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right]}{h}\cdot\frac{x^2(x+h)^2}{x^2(x+h)^2}$$
$$=\frac{x^2-(x+h)^2}{hx^2(x+h)^2}$$
$$=\frac{x^2-(x^2+2xh+h^2)}{hx^2(x+h)^2}$$
$$=\frac{-h(2x+h)}{hx^2(x+h)^2}$$
$$=-\frac{2x+h}{x^2(x+h)^2},\ h\neq 0$$

71. $$\frac{\left(\dfrac{1}{x+h-4}-\dfrac{1}{x-4}\right)}{h} = \frac{\left(\dfrac{1}{x+h-4}-\dfrac{1}{x-4}\right)}{h}\cdot\frac{(x-4)(x+h-4)}{(x-4)(x+h-4)}$$
$$= \frac{(x-4)-(x+h-4)}{h(x-4)(x+h-4)}$$
$$= \frac{-h}{h(x-4)(x+h-4)}$$
$$= -\frac{1}{(x-4)(x+h-4)}, \quad h \neq 0$$

72. $$\frac{\left(\dfrac{x+h}{x+h+1}-\dfrac{x}{x+1}\right)}{h} = \frac{\left(\dfrac{(x+h)(x+1)}{(x+h+1)(x+1)}-\dfrac{x(x+h+1)}{(x+h+1)(x+1)}\right)}{h/1}$$
$$= \left(\frac{(x+h)(x+1)}{(x+h+1)(x+1)}-\frac{x(x+h+1)}{(x+h+1)(x+1)}\right)\cdot\frac{1}{h}$$
$$= \left(\frac{x^2+x+hx+h-x^2-xh-x}{(x+h+1)(x+1)}\right)\cdot\frac{1}{h}$$
$$= \frac{h}{(x+h+1)(x+1)}\cdot\frac{1}{h} = \frac{1}{(x+h+1)(x+1)}, \; h \neq 0$$

73. $$\frac{\sqrt{x+2}-\sqrt{x}}{2} = \frac{\sqrt{x+2}-\sqrt{x}}{2}\cdot\frac{\sqrt{x+2}+\sqrt{x}}{\sqrt{x+2}+\sqrt{x}}$$
$$= \frac{(x+2)-x}{2\left(\sqrt{x+2}+\sqrt{x}\right)}$$
$$= \frac{2}{2\left(\sqrt{x+2}+\sqrt{x}\right)}$$
$$= \frac{1}{\sqrt{x+2}+\sqrt{x}}$$

74. $$\frac{\sqrt{z-3}-\sqrt{z}}{3} = \frac{\sqrt{z-3}-\sqrt{z}}{3}\cdot\frac{\sqrt{z-3}+\sqrt{z}}{\sqrt{z-3}+\sqrt{z}}$$
$$= \frac{(z-3)-z}{3\left(\sqrt{z-3}+\sqrt{z}\right)}$$
$$= \frac{-3}{3\left(\sqrt{z-3}+\sqrt{z}\right)}$$
$$= \frac{-1}{\sqrt{z-3}+\sqrt{z}}$$

75. $$\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h} = \frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\cdot\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}$$
$$= \frac{(x+h+1)-(x+1)}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$
$$= \frac{h}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$
$$= \frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}, \quad h \neq 0$$

76. $$\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}\cdot\frac{\sqrt{x+h-2}+\sqrt{x-2}}{\left(\sqrt{x+h-2}+\sqrt{x-2}\right)} = \frac{x+h-2-x+2}{h\left(\sqrt{x+h-2}+\sqrt{x-2}\right)}$$
$$= \frac{h}{h\left(\sqrt{x+h-2}+\sqrt{x-2}\right)}$$
$$= \frac{1}{\sqrt{x+h-2}+\sqrt{x-2}}, \quad h \neq 0$$

77. $$\text{Probability} = \frac{\text{Shaded area}}{\text{Total area}} = \frac{x(x/2)}{x(2x+1)} = \frac{x/2}{2x+1}\cdot\frac{2}{2} = \frac{x}{2(2x+1)}$$

78. Probability $= \dfrac{\text{Shaded area}}{\text{Total area}} = \dfrac{\frac{1}{2} \cdot \frac{4}{x}(x+2)(x+x+4)}{\frac{1}{2}(x+4)\left[(x+2)+\frac{4}{x}(x+2)\right]}$

$$= \frac{\frac{4(x+2)(2x+4)}{x}}{(x+4)(x+2)\left(1+\frac{4}{x}\right)} = \frac{\frac{4 \cdot 2(x+2)^2}{x}}{(x+4)(x+2)\left(1+\frac{4}{x}\right)}$$

$$= \frac{8(x+2)^2}{x} \cdot \frac{1}{(x+4)(x+2)\left(1+\frac{4}{x}\right)}$$

$$= \frac{8(x+2)^2}{(x+4)(x+2)(x+4)} = \frac{8(x+2)}{(x+4)^2}$$

79. (a) $\dfrac{1}{16}$ minute

(b) $x\left(\dfrac{1}{16}\right) = \dfrac{x}{16}$ minutes

(c) $\dfrac{60}{16} = \dfrac{15}{4}$ minutes

80. $\dfrac{t}{3} + \dfrac{t}{5} = \dfrac{5t+3t}{15} = \dfrac{8t}{15}$

81. (a) $r = \dfrac{\left(\frac{24[48(400)-16{,}000]}{48}\right)}{\left[16{,}000+\frac{48(400)}{12}\right]} \approx 0.0909 = 9.09\%$

(b) $r = \dfrac{\left[\frac{24(NM-P)}{N}\right]}{\left(P+\frac{NM}{12}\right)} = \dfrac{24(NM-P)}{N} \cdot \dfrac{12}{12P+NM} = \dfrac{288(NM-P)}{N(12P+NM)}$

$$r = \frac{288[48(400)-16{,}000]}{48[12(16{,}000)+48(400)]} \approx 0.0909 = 9.09\%$$

82. (a) $r = \dfrac{\left[\frac{24(NM-P)}{N}\right]}{\left(P+\frac{NM}{12}\right)} = \dfrac{\left[\frac{24(60 \cdot 525-28{,}000)}{60}\right]}{\left(28{,}000+\frac{60 \cdot 525}{12}\right)} = -\dfrac{\frac{2(31{,}500-28{,}000)}{5}}{28{,}000+\frac{31{,}500}{12}}$

$$= \frac{\frac{2(3500)}{5}}{28{,}000+2625} = \frac{2(700)}{30{,}625} = \frac{1400}{30{,}625} \approx 4.57\%$$

(b) $r = \dfrac{\left[\frac{24(NM-P)}{N}\right]}{\frac{12P+NM}{12}} = \dfrac{24(NM-P)}{N} \cdot \dfrac{12}{12P+NM} = \dfrac{288(NM-P)}{N(12P+NM)}$

$$= \frac{288(60 \cdot 525-28{,}000)}{60(12 \cdot 28{,}000+60 \cdot 525)} = \frac{288(31{,}500-28{,}000)}{60(336{,}000+31{,}500)} = \frac{288(3500)}{60(367{,}500)} = \frac{1{,}008{,}000}{22{,}050{,}000} \approx 4.57\%$$

83. $T = 10\left(\dfrac{4t^2 + 16t + 75}{t^2 + 4t + 10}\right)$

(a)

t	0	2	4	6	8	10	12	14	16	18	20	22
T	75°	55.9°	48.3°	45°	43.3°	42.3°	41.7°	41.3°	41.1°	40.9°	40.7°	40.6°

(b) T is approaching 40°.

84. (a)

Year	2002	2003	2004	2005	2006	2007
Banking	21.9	27.0	31.4	35.6	40.2	45.6
Paying Bills	13.9	17.8	21.5	25.0	28.1	31.0

(b) The estimates and actual values are quite close for banking. For paying bills, they are close for 2002–2004, but the model generally tends to overestimate the number.

(c) $$\frac{\dfrac{4.39t + 5.5}{0.002t^2 + 0.01t + 1.0}}{\dfrac{-0.728t^2 + 23.81t - 0.3}{-0.049t^2 + 0.61t + 1.0}} = \frac{4.39t + 5.5}{0.002t^2 + 0.01t + 1.0} \cdot \frac{-0.049t^2 + 0.61t + 1.0}{-0.728t^2 + 23.81t - 0.3}$$

$$= \frac{-0.21511t^3 + 2.6779t^2 + 4.39t - 0.2695t^2 + 3.355t + 5.5}{-0.001456t^4 + 0.04762t^3 - 0.0006t^2 - 72.8t^3 + 0.2381t^2 - 0.003t - 0.728t^2 + 23.81t - 0.3}$$

$$= \frac{-0.21511t^3 + 2.4084t^2 + 7.745t + 5.5}{-0.001456t^4 - 72.75238t^3 - 0.4905t^2 + 23.807t - 0.3}$$

(d)

Year	2002	2003	2004	2005	2006	2007
Paying Bills/Banking	0.63	0.66	0.69	0.70	0.70	0.68

The ratio is approximately $\frac{2}{3}$, and it appears to peak in 2005–2006.

85. False. In order for the simplified expression to be equivalent to the original expression, the domain of the simplified expression needs to be restricted. If n is even, $x \neq \pm 1$. If n is odd, $x \neq 1$.

86. False. The two expressions are equivalent for all values of x such that $x \neq 1$.

87. Completely factor the numerator and the denominator. A rational expression is in **simplest** form if there are no common factors in the numerator and the denominator other than ± 1.

Appendix A.5 Solving Equations

- You should know how to solve linear equations.
  $ax + b = 0$
- An identity is an equation whose solution consists of every real number in its domain.
- To solve an equation you can:
  (a) Add or subtract the same quantity from both sides.
  (b) Multiply or divide both sides by the same nonzero quantity.
- To solve an equation that can be simplified to a linear equation:
  (a) Remove all symbols of grouping and all fractions.
  (b) Combine like terms.
  (c) Solve by algebra.
  (d) Check the answer.
- A "solution" that does not satisfy the original equation is called an extraneous solution.
- You should be able to solve a quadratic equation by factoring, if possible.
- You should be able to solve a quadratic equation of the form $u^2 = d$ by extracting square roots.
- You should be able to solve a quadratic equation by completing the square.
- You should know and be able to use the Quadratic Formula: For $ax^2 + bx + c = 0$, $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- You should be able to solve polynomials of higher degree by factoring.
- For equations involving radicals or fractional powers, raise both sides to the same power.
- For equations with fractions, multiply both sides by the least common denominator to clear the fractions.
- For equations involving absolute value, remember that the expression inside the absolute value can be positive or negative.

Vocabulary Check

1. equation **2.** solve **3.** identities; conditional

4. $ax + b = 0$ **5.** extraneous **6.** quadratic equation

7. factoring; extracting square roots; completing the square; Quadratic Formula

1. $2(x - 1) = 2x - 2$ is an *identity* by the Distributive Property. It is true for all real values of x.

2. $3(x + 2) = 5x + 4$ is *conditional*. There are real values of x for which the equation is not true (for example, $x = 0$).

3. $-6(x - 3) + 5 = -2x + 10$ is *conditional*. There are real values of x for which the equation is not true.

4. $3(x + 2) - 5 = 3x + 1$ is an *identity* by simplification. It is true for all real values of x.

$3(x + 2) - 5 = 3x + 6 - 5 = 3x + 1$

5. $4(x + 1) - 2x = 4x + 4 - 2x = 2x + 4 = 2(x + 2)$

This is an *identity* by simplification. It is true for all real values of x.

6. $-7(x - 3) + 4x = 3(7 - x)$ is an *identity* by simplification. It is true for all real values of x.

$-7(x - 3) + 4x = -7x + 21 + 4x$

$= 21 - 3x = 3(7 - x)$

7. $(x-4)^2 - 11 = x^2 - 8x + 16 - 11 = x^2 - 8x + 5$

Thus, $x^2 - 8x + 5 = (x-4)^2 - 11$ is an *identity* by simplification. It is true for all real values of x.

8. $x^2 + 2(3x-2) = x^2 + 6x - 4$ is an *identity* by simplification. It is true for all real values of x.

9. $3 + \dfrac{1}{x+1} = \dfrac{4x}{x+1}$ is *conditional*. There are real values of x for which the equation is not true.

10. $\dfrac{5}{x} + \dfrac{3}{x} = 24$ is *conditional.* There are real values of x for which the equation is not true (for example, $x = 1$).

11.
$$x + 11 = 15$$
$$x + 11 - 11 = 15 - 11$$
$$x = 4$$

12.
$$7 - x = 19$$
$$7 - x + x = 19 + x$$
$$7 = 19 + x$$
$$7 - 19 = 19 + x - 19$$
$$-12 = x$$

13.
$$7 - 2x = 25$$
$$7 - 7 - 2x = 25 - 7$$
$$-2x = 18$$
$$\frac{-2x}{-2} = \frac{18}{-2}$$
$$x = -9$$

14.
$$7x + 2 = 23$$
$$7x + 2 - 2 = 23 - 2$$
$$7x = 21$$
$$\frac{7x}{7} = \frac{21}{7}$$
$$x = 3$$

15.
$$8x - 5 = 3x + 20$$
$$8x - 3x - 5 = 3x - 3x + 20$$
$$5x - 5 = 20$$
$$5x - 5 + 5 = 20 + 5$$
$$5x = 25$$
$$\frac{5x}{5} = \frac{25}{5}$$
$$x = 5$$

16.
$$7x + 3 = 3x - 17$$
$$7x + 3 - 3 - 3x = 3x - 17 - 3 - 3x$$
$$4x = -20$$
$$x = -5$$

17.
$$2(x+5) - 7 = 3(x-2)$$
$$2x + 10 - 7 = 3x - 6$$
$$2x + 3 = 3x - 6$$
$$2x - 3x + 3 = 3x - 3x - 6$$
$$-x + 3 = -6$$
$$-x + 3 - 3 = -6 - 3$$
$$-x = -9$$
$$x = 9$$

18.
$$3(x+3) = 5(1-x) - 1$$
$$3x + 9 = 5 - 5x - 1$$
$$3x + 9 = 4 - 5x$$
$$3x + 9 + 5x - 9 = 4 - 5x + 5x - 9$$
$$8x = -5$$
$$x = -\tfrac{5}{8}$$

19.
$$x - 3(2x+3) = 8 - 5x$$
$$x - 6x - 9 = 8 - 5x$$
$$-5x - 9 = 8 - 5x$$
$$-5x + 5x - 9 = 8 - 5x + 5x$$
$$-9 \neq 8$$

No solution

20.
$$9x - 10 = 5x + 2(2x - 5)$$
$$9x - 10 = 5x + 4x - 10$$
$$9x - 10 = 9x - 10$$

The solution is the set of all real numbers.

21.
$$\frac{5x}{4} + \frac{1}{2} = x - \frac{1}{2}$$
$$4\left(\frac{5x}{4} + \frac{1}{2}\right) = 4\left(x - \frac{1}{2}\right)$$
$$4\left(\frac{5x}{4}\right) + 4\left(\frac{1}{2}\right) = 4(x) - 4\left(\frac{1}{2}\right)$$
$$5x + 2 = 4x - 2$$
$$x = -4$$

22.
$$\frac{x}{5} - \frac{x}{2} = 3 + \frac{3x}{10}$$
$$10\left(\frac{x}{5} - \frac{x}{2}\right) = 10\left(3 + \frac{3x}{10}\right)$$
$$2x - 5x = 30 + 3x$$
$$-6x = 30$$
$$x = -5$$

23. $\frac{3}{2}(z+5)-\frac{1}{4}(z+24)=0$

$4\left[\frac{3}{2}(z+5)-\frac{1}{4}(z+24)\right]=4(0)$

$4\left(\frac{3}{2}\right)(z+5)-4\left(\frac{1}{4}\right)(z+24)=4(0)$

$6(z+5)-(z+24)=0$

$6z+30-z-24=0$

$5z=-6$

$z=-\frac{6}{5}$

24. $\frac{3x}{2}+\frac{1}{4}(x-2)=10$

$4\left[\frac{3x}{2}+\frac{1}{4}(x-2)\right]=4(10)$

$4\left(\frac{3x}{2}\right)+4\left(\frac{1}{4}\right)(x-2)=4(10)$

$6x+(x-2)=40$

$7x-2=40$

$7x=42$

$x=6$

25. $0.25x+0.75(10-x)=3$

$0.25x+7.5-0.75x=3$

$-0.50x+7.5=3$

$-0.50x=-4.5$

$x=9$

26. $0.60x+0.40(100-x)=50$

$0.60x+40-0.40x=50$

$0.20x=10$

$x=50$

27. $x+8=2(x-2)-x$

$x+8=2x-4-x$

$x+8=x-4$

$8\neq -4$

Contradiction; no solution

28. $8(x+2)-3(2x+1)=2(x+5)$

$8x+16-6x-3=2x+10$

$2x+13=2x+10$

$13=10$

Contradiction; no solution

29. $\frac{100-4x}{3}=\frac{5x+6}{4}+6$

$12\left(\frac{100-4x}{3}\right)=12\left(\frac{5x+6}{4}\right)+12(6)$

$4(100-4x)=3(5x+6)+72$

$400-16x=15x+18+72$

$-31x=-310$

$x=10$

30. $\frac{17+y}{y}+\frac{32+y}{y}=100$

$(y)\frac{17+y}{y}+(y)\frac{32+y}{y}=100(y)$

$17+y+32+y=100y$

$49+2y=100y$

$49=98y$

$\frac{1}{2}=y$

31. $\frac{5x-4}{5x+4}=\frac{2}{3}$

$3(5x-4)=2(5x+4)$

$15x-12=10x+8$

$5x=20$

$x=4$

32. $\frac{10x+3}{5x+6}=\frac{1}{2}$

$2(10x+3)=1(5x+6)$

$20x+6=5x+6$

$15x=0$

$x=0$

33. $10 - \frac{13}{x} = 4 + \frac{5}{x}$

$$\frac{10x - 13}{x} = \frac{4x + 5}{x}$$
$$10x - 13 = 4x + 5$$
$$6x = 18$$
$$x = 3$$

34. $\frac{15}{x} - 4 = \frac{6}{x} + 3$

$$\frac{15}{x} - \frac{6}{x} = 7$$
$$\frac{9}{x} = 7$$
$$9 = 7x$$
$$\frac{9}{7} = x$$

35. $3 = 2 + \frac{2}{z + 2}$

$$3(z + 2) = \left(2 + \frac{2}{z + 2}\right)(z + 2)$$
$$3z + 6 = 2z + 4 + 2$$
$$z = 0$$

36. $\frac{1}{x} + \frac{2}{x - 5} = 0$ Multiply both sides by $x(x - 5)$.

$$1(x - 5) + 2x = 0$$
$$3x - 5 = 0$$
$$3x = 5$$
$$x = \frac{5}{3}$$

37. $\frac{x}{x + 4} + \frac{4}{x + 4} + 2 = 0$

$$\frac{x + 4}{x + 4} + 2 = 0$$
$$1 + 2 = 0$$
$$3 \neq 0$$

Contradiction; no solution

38. $\frac{7}{2x + 1} - \frac{8x}{2x - 1} = -4$ Multiply both sides by $(2x + 1)(2x - 1)$.

$$7(2x - 1) - 8x(2x + 1) = -4(2x + 1)(2x - 1)$$
$$14x - 7 - 16x^2 - 8x = -16x^2 + 4$$
$$6x = 11$$
$$x = \frac{11}{6}$$

39. $\frac{2}{(x - 4)(x - 2)} = \frac{1}{x - 4} + \frac{2}{x - 2}$ Multiply both sides by $(x - 4)(x - 2)$.

$$2 = 1(x - 2) + 2(x - 4)$$
$$2 = x - 2 + 2x - 8$$
$$2 = 3x - 10$$
$$12 = 3x$$
$$4 = x$$

A check reveals that $x = 4$ is an extraneous solution—it makes the denominator zero. There is no real solution.

40. $\frac{4}{x - 1} + \frac{6}{3x + 1} = \frac{15}{3x + 1}$ Multiply both sides by $(x - 1)(3x + 1)$.

$$(x - 1)(3x + 1)\frac{4}{x - 1} + (x - 1)(3x + 1)\frac{6}{3x + 1} = (x - 1)(3x + 1)\frac{15}{3x + 1}$$
$$4(3x + 1) + 6(x - 1) = 15(x - 1)$$
$$12x + 4 + 6x - 6 = 15x - 15$$
$$18x - 2 = 15x - 15$$
$$3x = -13$$
$$x = -\frac{13}{3}$$

41. $\dfrac{1}{x-3}+\dfrac{1}{x+3}=\dfrac{10}{x^2-9}$

$\dfrac{1}{x-3}+\dfrac{1}{x+3}=\dfrac{10}{(x+3)(x-3)}$ Multiply both sides by $(x+3)(x-3)$.

$1(x+3)+1(x-3)=10$

$2x=10$

$x=5$

42. $\dfrac{1}{x-2}+\dfrac{3}{x+3}=\dfrac{4}{x^2+x-6}$

$\dfrac{1}{x-2}+\dfrac{3}{x+3}=\dfrac{4}{(x+3)(x-2)}$ Multiply both sides by $(x+3)(x-2)$.

$(x+3)+3(x-2)=4$

$x+3+3x-6=4$

$4x-3=4$

$4x=7$

$x=\dfrac{7}{4}$

43. $\dfrac{3}{x^2-3x}+\dfrac{4}{x}=\dfrac{1}{x-3}$

$\dfrac{3}{x(x-3)}+\dfrac{4}{x}=\dfrac{1}{x-3}$ Multiply both sides by $x(x-3)$.

$3+4(x-3)=x$

$3+4x-12=x$

$3x=9$

$x=3$

A check reveals that $x=3$ is an extraneous solution since it makes the denominator zero, so there is no solution.

44. $\dfrac{6}{x}-\dfrac{2}{x+3}=\dfrac{3(x+5)}{x(x+3)}$ Multiply both sides by $x(x+3)$.

$6(x+3)-2x=3(x+5)$

$6x+18-2x=3x+15$

$4x+18=3x+15$

$x=-3$

Check: $\dfrac{6}{-3}-\dfrac{2}{-3+3}=\dfrac{3(-3+5)}{-3(-3+3)}$

$-2-\dfrac{2}{0}=\dfrac{-6}{-3(0)}$

Division by zero is undefined. Thus, $x=-3$ is not a solution, and the original equation has no solution.

45. $(x+2)^2+5=(x+3)^2$

$x^2+4x+4+5=x^2+6x+9$

$4x+9=6x+9$

$-2x=0$

$x=0$

46. $(x+1)^2+2(x-2)=(x+1)(x-2)$

$x^2+2x+1+2x-4=x^2-x-2$

$5x=1$

$x=\frac{1}{5}$

47. $(x + 2)^2 - x^2 = 4(x + 1)$

$x^2 + 4x + 4 - x^2 = 4x + 4$

$4 = 4$

The equation is an identity; every real number is a solution.

48. $(2x + 1)^2 = 4(x^2 + x + 1)$

$4x^2 + 4x + 1 = 4x^2 + 4x + 4$

$1 = 4$

This is a contradiction. Thus, the equation has no solution.

49. $2x^2 = 3 - 8x$

General form: $2x^2 + 8x - 3 = 0$

50. $x^2 = 16x$

General form: $x^2 - 16x = 0$

51. $(x - 3)^2 = 3$

$x^2 - 6x + 9 = 3$

General form: $x^2 - 6x + 6 = 0$

52. $13 - 3(x + 7)^2 = 0$

$13 - 3(x^2 + 14x + 49) = 0$

$13 - 3x^2 - 42x - 147 = 0$

General form:

$-3x^2 - 42x - 134 = 0$

53. $\frac{1}{5}(3x^2 - 10) = 18x$

$3x^2 - 10 = 90x$

General form: $3x^2 - 90x - 10 = 0$

54. $x(x + 2) = 5x^2 + 1$

$x^2 + 2x = 5x^2 + 1$

$-4x^2 + 2x - 1 = 0$

$(-1)(-4x^2 + 2x - 1) = -1(0)$

General form: $4x^2 - 2x + 1 = 0$

55. $6x^2 + 3x = 0$

$3x(2x + 1) = 0$

$3x = 0$ or $2x + 1 = 0$

$x = 0$ or $x = -\frac{1}{2}$

56. $9x^2 - 1 = 0$

$(3x + 1)(3x - 1) = 0$

$3x + 1 = 0 \Rightarrow x = -\frac{1}{3}$

$3x - 1 = 0 \Rightarrow x = \frac{1}{3}$

57. $x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x - 4 = 0$ or $x + 2 = 0$

$x = 4$ or $x = -2$

58. $x^2 - 10x + 9 = 0$

$(x - 9)(x - 1) = 0$

$x - 9 = 0 \Rightarrow x = 9$

$x - 1 = 0 \Rightarrow x = 1$

59. $x^2 + 10x + 25 = 0$

$(x + 5)^2 = 0$

$x + 5 = 0$

$x = -5$

60. $4x^2 + 12x + 9 = 0$

$(2x + 3)(2x + 3) = 0$

$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

61. $3 + 5x - 2x^2 = 0$

$(3 - x)(1 + 2x) = 0$

$3 - x = 0$ or $1 + 2x = 0$

$x = 3$ or $x = -\frac{1}{2}$

62. $2x^2 = 19x + 33$

$2x^2 - 19x - 33 = 0$

$(2x + 3)(x - 11) = 0$

$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

$x - 11 = 0 \Rightarrow x = 11$

63. $x^2 + 4x = 12$

$x^2 + 4x - 12 = 0$

$(x + 6)(x - 2) = 0$

$x + 6 = 0$ or $x - 2 = 0$

$x = -6$ or $x = 2$

64. $-x^2 + 8x = 12$

$-x^2 + 8x - 12 = 0$

$(-1)(-x^2 + 8x - 12) = (-1)(0)$

$x^2 - 8x + 12 = 0$

$(x - 6)(x - 2) = 0$

$x - 6 = 0 \Rightarrow x = 6$

$x - 2 = 0 \Rightarrow x = 2$

65. $\frac{3}{4}x^2 + 8x + 20 = 0$

$4\left(\frac{3}{4}x^2 + 8x + 20\right) = 4(0)$

$3x^2 + 32x + 80 = 0$

$(3x + 20)(x + 4) = 0$

$3x + 20 = 0$ or $x + 4 = 0$

$x = -\frac{20}{3}$ or $x = -4$

66. $\frac{1}{8}x^2 - x - 16 = 0$

$x^2 - 8x - 128 = 0$

$(x - 16)(x + 8) = 0$

$x - 16 = 0 \Rightarrow x = 16$

$x + 8 = 0 \Rightarrow x = -8$

67. $x^2 + 2ax + a^2 = 0$

$(x + a)^2 = 0$

$x + a = 0$

$x = -a$

68. $(x + a)^2 - b^2 = 0$

$[(x + a) + b][(x + a) - b] = 0$

$[x + (a + b)][x + (a - b)] = 0$

$x + (a + b) = 0 \Rightarrow x = -a - b$

$x + (a - b) = 0 \Rightarrow x = -a + b$

69. $x^2 = 49$

$x = \pm 7$

70. $x^2 = 169$

$x = \pm\sqrt{169} = \pm 13$

71. $x^2 = 11$

$x = \pm\sqrt{11}$

72. $x^2 = 32$

$x = \pm\sqrt{32} = \pm 4\sqrt{2}$

73. $3x^2 = 81$

$x^2 = 27$

$x = \pm 3\sqrt{3}$

74. $9x^2 = 36$

$x^2 = 4$

$x = \pm\sqrt{4} = \pm 2$

75. $(x - 12)^2 = 16$

$x - 12 = \pm 4$

$x = 12 \pm 4$

$x = 16$ or $x = 8$

76. $(x + 13)^2 = 25$

$x + 13 = \pm\sqrt{25}$

$x + 13 = \pm 5$

$x = -13 \pm 5 = -8, -18$

77. $(x + 2)^2 = 14$

$x + 2 = \pm\sqrt{14}$

$x = -2 \pm \sqrt{14}$

78. $(x - 5)^2 = 30$

$x - 5 = \pm\sqrt{30}$

$x = 5 \pm \sqrt{30}$

79. $(2x - 1)^2 = 18$

$2x - 1 = \pm\sqrt{18}$

$2x = 1 \pm 3\sqrt{2}$

$x = \dfrac{1 \pm 3\sqrt{2}}{2}$

80. $(4x + 7)^2 = 44$

$4x + 7 = \pm\sqrt{44}$

$4x = -7 \pm 2\sqrt{11}$

$x = \dfrac{-7 \pm 2\sqrt{11}}{4} = -\dfrac{7}{4} \pm \dfrac{\sqrt{11}}{2}$

81. $(x - 7)^2 = (x + 3)^2$

$x - 7 = \pm(x + 3)$

$x - 7 = x + 3$ or $x - 7 = -x - 3$

$-7 \neq 3$ or $2x = 4$

$x = 2$

The only solution to the equation is $x = 2$.

82. $(x + 5)^2 = (x + 4)^2$

$x + 5 = \pm(x + 4)$

$x + 5 = +(x + 4)$ or $x + 5 = -(x + 4)$

$5 \neq 4$ or $x + 5 = -x - 4$

$2x = -9$

$x = -\frac{9}{2}$

The only solution to the equation is $x = -\frac{9}{2}$.

83. $x^2 + 4x - 32 = 0$

$x^2 + 4x = 32$

$x^2 + 4x + 2^2 = 32 + 2^2$

$(x + 2)^2 = 36$

$x + 2 = \pm 6$

$x = -2 \pm 6$

$x = 4$ or $x = -8$

84. $x^2 - 2x - 3 = 0$

$x^2 - 2x = 3$

$x^2 - 2x + (-1)^2 = 3 + (-1)^2$

$(x - 1)^2 = 4$

$x - 1 = \pm\sqrt{4}$

$x = 1 \pm 2$

$x = 3$ or $x = -1$

85. $x^2 + 12x + 25 = 0$

$x^2 + 12x = -25$

$x^2 + 12x + 6^2 = -25 + 6^2$

$(x + 6)^2 = 11$

$x + 6 = \pm\sqrt{11}$

$x = -6 \pm \sqrt{11}$

86. $x^2 + 8x + 14 = 0$

$x^2 + 8x = -14$

$x^2 + 8x + 4^2 = -14 + 16$

$(x + 4)^2 = 2$

$x + 4 = \pm\sqrt{2}$

$x = -4 \pm \sqrt{2}$

87. $9x^2 - 18x = -3$

$x^2 - 2x = -\frac{1}{3}$

$x^2 - 2x + 1^2 = -\frac{1}{3} + 1^2$

$(x - 1)^2 = \frac{2}{3}$

$x - 1 = \pm\sqrt{\frac{2}{3}}$

$x = 1 \pm \sqrt{\frac{2}{3}}$

$x = 1 \pm \frac{\sqrt{6}}{3}$

88. $9x^2 - 12x = 14$

$x^2 - \frac{4}{3}x = \frac{14}{9}$

$x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 = \frac{14}{9} + \frac{4}{9}$

$\left(x - \frac{2}{3}\right)^2 = \frac{18}{9}$

$\left(x - \frac{2}{3}\right)^2 = 2$

$x - \frac{2}{3} = \pm\sqrt{2}$

$x = \frac{2}{3} \pm \sqrt{2}$

89. $8 + 4x - x^2 = 0$

$-x^2 + 4x + 8 = 0$

$x^2 - 4x - 8 = 0$

$x^2 - 4x = 8$

$x^2 - 4x + 2^2 = 8 + 2^2$

$(x - 2)^2 = 12$

$x - 2 = \pm\sqrt{12}$

$x = 2 \pm 2\sqrt{3}$

90. $-x^2 + x - 1 = 0$

$x^2 - x + 1 = 0$

$x^2 - x + \frac{1}{4} = -1 + \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 = -\frac{3}{4}$

No real solution

91. $2x^2 + 5x - 8 = 0$

$2x^2 + 5x = 8$

$x^2 + \frac{5}{2}x = 4$

$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = 4 + \left(\frac{5}{4}\right)^2$

$\left(x + \frac{5}{4}\right)^2 = \frac{89}{16}$

$x + \frac{5}{4} = \pm\frac{\sqrt{89}}{4}$

$x = -\frac{5}{4} \pm \frac{\sqrt{89}}{4}$

$x = \frac{-5 \pm \sqrt{89}}{4}$

92. $4x^2 - 4x - 99 = 0$

$x^2 - x = \frac{99}{4}$

$x^2 - x + \left(-\frac{1}{2}\right)^2 = \frac{99}{4} + \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 = \frac{100}{4}$

$\left(x - \frac{1}{2}\right)^2 = 25$

$x - \frac{1}{2} = \pm\sqrt{25}$

$x = \frac{1}{2} \pm 5 = \frac{11}{2}, -\frac{9}{2}$

93. $2x^2 + x - 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)}$

$= \frac{-1 \pm 3}{4} = \frac{1}{2}, -1$

94. $2x^2 - x - 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)}$

$= \frac{1 \pm \sqrt{1 + 8}}{4}$

$= \frac{1 \pm 3}{4} = 1, -\frac{1}{2}$

95. $16x^2 + 8x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{8^2 - 4(16)(-3)}}{2(16)}$$

$$= \frac{-8 \pm 16}{32} = \frac{1}{4}, -\frac{3}{4}$$

96. $25x^2 - 20x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(25)(3)}}{2(25)}$$

$$= \frac{20 \pm \sqrt{400 - 300}}{50}$$

$$= \frac{20 \pm 10}{50} = \frac{3}{5}, \frac{1}{5}$$

97. $2 + 2x - x^2 = 0$

$$-x^2 + 2x + 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)}$$

$$= \frac{-2 \pm 2\sqrt{3}}{-2} = 1 \pm \sqrt{3}$$

98. $x^2 - 10x + 22 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(22)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{100 - 88}}{2}$$

$$= \frac{10 \pm 2\sqrt{3}}{2} = 5 \pm \sqrt{3}$$

99. $x^2 + 14x + 44 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-14 \pm \sqrt{14^2 - 4(1)(44)}}{2(1)}$$

$$= \frac{-14 \pm 2\sqrt{5}}{2} = -7 \pm \sqrt{5}$$

100. $6x = 4 - x^2$

$$x^2 + 6x - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{6^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 + 16}}{2}$$

$$= \frac{-6 \pm 2\sqrt{13}}{2}$$

$$= -3 \pm \sqrt{13}$$

101. $x^2 + 8x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{8^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-8 \pm 4\sqrt{5}}{2} = -4 \pm 2\sqrt{5}$$

102. $4x^2 - 4x - 4 = 0$

$$x^2 - x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 + 4}}{2}$$

$$= \frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

103. $12x - 9x^2 = -3$

$$-9x^2 + 12x + 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-12 \pm \sqrt{12^2 - 4(-9)(3)}}{2(-9)}$$

$$= \frac{-12 \pm 6\sqrt{7}}{-18} = \frac{2}{3} \pm \frac{\sqrt{7}}{3}$$

104. $16x^2 + 22 = 40x$

$$8x^2 - 20x + 11 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(8)(11)}}{2(8)}$$

$$= \frac{20 \pm \sqrt{400 - 352}}{16}$$

$$= \frac{5}{4} \pm \frac{\sqrt{3}}{4}$$

105. $9x^2 + 24x + 16 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-24 \pm \sqrt{24^2 - 4(9)(16)}}{2(9)}$$

$$= \frac{-24 \pm 0}{18}$$

$$= -\frac{4}{3}$$

106. $36x^2 + 24x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-24 \pm \sqrt{24^2 - 4(36)(-7)}}{2(36)}$$

$$= \frac{-24 \pm \sqrt{576 + 1008}}{72}$$

$$= \frac{-24 \pm \sqrt{(144)(11)}}{72}$$

$$= -\frac{1}{3} \pm \frac{\sqrt{11}}{6}$$

107. $4x^2 + 4x = 7$

$4x^2 + 4x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(4)(-7)}}{2(4)}$$

$$= \frac{-4 \pm 8\sqrt{2}}{8} = -\frac{1}{2} \pm \sqrt{2}$$

108. $16x^2 - 40x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-40) \pm \sqrt{(-40)^2 - 4(16)(5)}}{2(16)}$$

$$= \frac{40 \pm \sqrt{1600 - 320}}{32}$$

$$= \frac{40 \pm 16\sqrt{5}}{32}$$

$$= \frac{5}{4} \pm \frac{\sqrt{5}}{2}$$

109. $28x - 49x^2 = 4$

$-49x^2 + 28x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-28 \pm \sqrt{28^2 - 4(-49)(-4)}}{2(-49)}$$

$$= \frac{-28 \pm 0}{-98} = \frac{2}{7}$$

110. $3x + x^2 - 1 = 0$

$x^2 + 3x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{13}}{2} = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

111. $8t = 5 + 2t^2$

$-2t^2 + 8t - 5 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{8^2 - 4(-2)(-5)}}{2(-2)}$$

$$= \frac{-8 \pm 2\sqrt{6}}{-4} = 2 \pm \frac{\sqrt{6}}{2}$$

112. $25h^2 + 80h + 61 = 0$

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-80 \pm \sqrt{80^2 - 4(25)(61)}}{2(25)}$$

$$= \frac{-80 \pm \sqrt{6400 - 6100}}{50}$$

$$= -\frac{8}{5} \pm \frac{10\sqrt{3}}{50}$$

$$= -\frac{8}{5} \pm \frac{\sqrt{3}}{5}$$

113. $(y - 5)^2 = 2y$

$y^2 - 12y + 25 = 0$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{12 \pm 2\sqrt{11}}{2} = 6 \pm \sqrt{11}$$

114. $(z + 6)^2 = -2z$

$z^2 + 12z + 36 = -2z$

$z^2 + 14z + 36 = 0$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-14 \pm \sqrt{14^2 - 4(1)(36)}}{2(1)}$$

$$= \frac{-14 \pm \sqrt{52}}{2}$$

$$= -7 \pm \sqrt{13}$$

115. $\frac{1}{2}x^2 + \frac{3}{8}x = 2$

$4x^2 + 3x = 16$

$4x^2 + 3x - 16 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(4)(-16)}}{2(4)}$$

$$= \frac{-3 \pm \sqrt{265}}{8}$$

$$= -\frac{3}{8} \pm \frac{\sqrt{265}}{8}$$

116. $\left(\frac{5}{7}x - 14\right)^2 = 8x$

$\frac{25}{49}x^2 - 20x + 196 = 8x$

$\frac{25}{49}x^2 - 28x + 196 = 0$

$25x^2 - 1372x + 9604 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1372) \pm \sqrt{(-1372)^2 - 4(25)(9604)}}{2(25)} = \frac{1372 \pm \sqrt{921{,}984}}{50} = \frac{686 \pm 196\sqrt{6}}{25}$$

117. $5.1x^2 - 1.7x - 3.2 = 0$

$$x = \frac{1.7 \pm \sqrt{(-1.7)^2 - 4(5.1)(-3.2)}}{2(5.1)}$$

$$x \approx 0.976, -0.643$$

118. $2x^2 - 2.50x - 0.42 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2.50) \pm \sqrt{(-2.50)^2 - 4(2)(-0.42)}}{2(2)}$$

$$= \frac{2.50 \pm \sqrt{9.61}}{4}$$

$$= 1.400, -0.150$$

119. $-0.067x^2 - 0.852x + 1.277 = 0$

$$x = \frac{-(-0.852) \pm \sqrt{(-0.852)^2 - 4(-0.067)(1.277)}}{2(-0.067)}$$

$$x \approx -14.071, 1.355$$

120. $-0.005x^2 + 0.101x - 0.193 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-0.101 \pm \sqrt{(0.101)^2 - 4(-0.005)(-0.193)}}{2(-0.005)}$$

$$= \frac{-0.101 \pm \sqrt{0.006341}}{-0.01}$$

$$\approx 2.137, 18.063$$

121. $422x^2 - 506x - 347 = 0$

$$x = \frac{506 \pm \sqrt{(-506)^2 - 4(422)(-347)}}{2(422)}$$

$$x \approx 1.687, -0.488$$

122. $1100x^2 + 326x - 715 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-326 \pm \sqrt{(326)^2 - 4(1100)(-715)}}{2(1100)}$$

$$= \frac{-326 \pm \sqrt{3{,}252{,}276}}{2200} \approx 0.672, -0.968$$

123. $12.67x^2 + 31.55x + 8.09 = 0$

$$x = \frac{-31.55 \pm \sqrt{(31.55)^2 - 4(12.67)(8.09)}}{2(12.67)}$$

$$x \approx -2.200, -0.290$$

124. $-3.22x^2 - 0.08x + 28.651 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-0.08) \pm \sqrt{(-0.08)^2 - 4(-3.22)(28.651)}}{2(-3.22)}$$

$$= \frac{0.08 \pm \sqrt{369.031}}{-6.44} \approx -2.995, 2.971$$

125. $x^2 - 2x - 1 = 0$ Complete the square.

$$x^2 - 2x = 1$$

$$x^2 - 2x + 1^2 = 1 + 1^2$$

$$(x - 1)^2 = 2$$

$$x - 1 = \pm\sqrt{2}$$

$$x = 1 \pm \sqrt{2}$$

126. $11x^2 + 33x = 0$ Factor.

$$11(x^2 + 3x) = 0$$

$$x(x + 3) = 0$$

$$x = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = -3$$

127. $(x + 3)^2 = 81$ Extract square roots.

$$x + 3 = \pm 9$$

$$x + 3 = 9 \quad \text{or} \quad x + 3 = -9$$

$$x = 6 \quad \text{or} \quad x = -12$$

128. $x^2 - 14x + 49 = 0$ Extract square roots.

$$(x - 7)^2 = 0$$

$$x - 7 = 0$$

$$x = 7$$

129. $x^2 - x - \frac{11}{4} = 0$ Complete the square.

$$x^2 - x = \frac{11}{4}$$
$$x^2 - x + \left(\frac{1}{2}\right)^2 = \frac{11}{4} + \left(\frac{1}{2}\right)^2$$
$$\left(x - \frac{1}{2}\right)^2 = \frac{12}{4}$$
$$x - \frac{1}{2} = \pm\sqrt{\frac{12}{4}}$$
$$x = \frac{1}{2} \pm \sqrt{3}$$

130. $x^2 + 3x - \frac{3}{4} = 0$ Complete the square.

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = \frac{3}{4} + \frac{9}{4}$$
$$\left(x + \frac{3}{2}\right)^2 = 3$$
$$x + \frac{3}{2} = \pm\sqrt{3}$$
$$x = -\frac{3}{2} \pm \sqrt{3}$$

131. $(x + 1)^2 = x^2$ Extract square roots.

$$x^2 = (x + 1)^2$$
$$x = \pm(x + 1)$$

For $x = +(x + 1)$:

$0 \neq 1$ No solution

For $x = -(x + 1)$:

$$2x = -1$$
$$x = -\frac{1}{2}$$

132. $a^2x^2 - b^2 = 0$ Factor.

$$(ax + b)(ax - b) = 0$$
$$ax + b = 0 \implies x = -\frac{b}{a}$$
$$ax - b = 0 \implies x = \frac{b}{a}$$

133. $3x + 4 = 2x^2 - 7$ Quadratic Formula

$$0 = 2x^2 - 3x - 11$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-11)}}{2(2)}$$
$$= \frac{3 \pm \sqrt{97}}{4}$$
$$= \frac{3}{4} \pm \frac{\sqrt{97}}{4}$$

134. $4x^2 + 2x + 4 = 2x + 8$ Factor.

$$4x^2 - 4 = 0$$
$$4(x^2 - 1) = 0$$
$$(x + 1)(x - 1) = 0$$
$$x + 1 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -1 \qquad x = 1$$

135. $4x^4 - 18x^2 = 0$

$$2x^2(2x^2 - 9) = 0$$
$$2x^2 = 0 \implies x = 0$$
$$2x^2 - 9 = 0 \implies x = \pm\frac{3\sqrt{2}}{2}$$

136. $20x^3 - 125x = 0$

$$5x(4x^2 - 25) = 0$$
$$5x(2x + 5)(2x - 5) = 0$$
$$5x = 0 \implies x = 0$$
$$2x + 5 = 0 \implies x = -\frac{5}{2}$$
$$2x - 5 = 0 \implies x = \frac{5}{2}$$

137. $x^4 - 81 = 0$

$$(x^2 + 9)(x + 3)(x - 3) = 0$$
$$x^2 + 9 = 0 \implies \text{No real solution}$$
$$x + 3 = 0 \implies x = -3$$
$$x - 3 = 0 \implies x = 3$$

138.
$$x^6 - 64 = 0$$
$$(x^3 - 8)(x^3 + 8) = 0$$
$$(x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4) = 0$$
$$x - 2 = 0 \Rightarrow x = 2$$
$$x^2 + 2x + 4 = 0 \Rightarrow \text{No real solution (by the Quadratic Formula)}$$
$$x + 2 = 0 \Rightarrow x = -2$$
$$x^2 - 2x + 4 = 0 \Rightarrow \text{No real solution (by the Quadratic Formula)}$$

139.
$$x^3 + 216 = 0$$
$$x^3 + 6^3 = 0$$
$$(x + 6)(x^2 - 6x + 36) = 0$$
$$x + 6 = 0 \Rightarrow x = -6$$
$$x^2 - 6x + 36 = 0 \Rightarrow \text{No real solution (by the Quadratic Formula)}$$

140.
$$27x^3 - 512 = 0$$
$$(3x - 8)(9x^2 + 24x + 64) = 0$$
$$3x - 8 = 0 \Rightarrow x = \tfrac{8}{3}$$
$$9x^2 + 24x + 64 = 0 \Rightarrow \text{No real solution (by the Quadratic Formula)}$$

141.
$$5x^3 + 30x^2 + 45x = 0$$
$$5x(x^2 + 6x + 9) = 0$$
$$5x(x + 3)^2 = 0$$
$$5x = 0 \Rightarrow x = 0$$
$$x + 3 = 0 \Rightarrow x = -3$$

142.
$$9x^4 - 24x^3 + 16x^2 = 0$$
$$x^2(9x^2 - 24x + 16) = 0$$
$$x^2(3x - 4)^2 = 0$$
$$x^2 = 0 \Rightarrow x = 0$$
$$3x - 4 = 0 \Rightarrow x = \tfrac{4}{3}$$

143.
$$x^3 - 3x^2 - x + 3 = 0$$
$$x^2(x - 3) - (x - 3) = 0$$
$$(x - 3)(x^2 - 1) = 0$$
$$(x - 3)(x + 1)(x - 1) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x - 1 = 0 \Rightarrow x = 1$$

144.
$$x^3 + 2x^2 + 3x + 6 = 0$$
$$x^2(x + 2) + 3(x + 2) = 0$$
$$(x + 2)(x^2 + 3) = 0$$
$$x + 2 = 0 \Rightarrow x = -2$$
$$x^2 + 3 = 0 \Rightarrow \text{No real solution}$$

145.
$$x^4 - x^3 + x - 1 = 0$$
$$x^3(x - 1) + (x - 1) = 0$$
$$(x - 1)(x^3 + 1) = 0$$
$$(x - 1)(x + 1)(x^2 - x + 1) = 0$$
$$x - 1 = 0 \Rightarrow x = 1$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x^2 - x + 1 = 0 \Rightarrow \text{No real solution (by the Quadratic Formula)}$$

146.
$$x^4 + 2x^3 - 8x - 16 = 0$$
$$x^3(x + 2) - 8(x + 2) = 0$$
$$(x^3 - 8)(x + 2) = 0$$
$$(x - 2)(x^2 + 2x + 4)(x + 2) = 0$$
$$x - 2 = 0 \implies x = 2$$
$$x^2 + 2x + 4 = 0 \implies \text{No real solution (by the Quadratic Formula)}$$
$$x + 2 = 0 \implies x = -2$$

147.
$$x^4 - 4x^2 + 3 = 0$$
$$(x^2 - 3)(x^2 - 1) = 0$$
$$\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)(x + 1)(x - 1) = 0$$
$$x + \sqrt{3} = 0 \implies x = -\sqrt{3}$$
$$x - \sqrt{3} = 0 \implies x = \sqrt{3}$$
$$x + 1 = 0 \implies x = -1$$
$$x - 1 = 0 \implies x = 1$$

148.
$$x^4 + 5x^2 - 36 = 0$$
$$(x^2 + 9)(x^2 - 4) = 0$$
$$(x^2 + 9)(x + 2)(x - 2) = 0$$
$$x^2 + 9 = 0 \implies \text{No real solution}$$
$$x + 2 = 0 \implies x = -2$$
$$x - 2 = 0 \implies x = 2$$

149.
$$4x^4 - 65x^2 + 16 = 0$$
$$(4x^2 - 1)(x^2 - 16) = 0$$
$$(2x + 1)(2x - 1)(x + 4)(x - 4) = 0$$
$$2x + 1 = 0 \implies x = -\tfrac{1}{2}$$
$$2x - 1 = 0 \implies x = \tfrac{1}{2}$$
$$x + 4 = 0 \implies x = -4$$
$$x - 4 = 0 \implies x = 4$$

150.
$$36t^4 + 29t^2 - 7 = 0$$
$$(36t^2 - 7)(t^2 + 1) = 0$$
$$\left(6t + \sqrt{7}\right)\left(6t - \sqrt{7}\right)(t^2 + 1) = 0$$
$$6t + \sqrt{7} = 0 \implies t = -\frac{\sqrt{7}}{6}$$
$$6t - \sqrt{7} = 0 \implies t = \frac{\sqrt{7}}{6}$$
$$t^2 + 1 = 0 \implies \text{No real solution}$$

151.
$$x^6 + 7x^3 - 8 = 0$$
$$(x^3 + 8)(x^3 - 1) = 0$$
$$(x + 2)(x^2 - 2x + 4)(x - 1)(x^2 + x + 1) = 0$$
$$x + 2 = 0 \implies x = -2$$
$$x^2 - 2x + 4 = 0 \implies \text{No real solution (by the Quadratic Formula)}$$
$$x - 1 = 0 \implies x = 1$$
$$x^2 + x + 1 = 0 \implies \text{No real solution (by the Quadratic Formula)}$$

152.
$$x^6 + 3x^3 + 2 = 0$$
$$(x^3 + 2)(x^3 + 1) = 0$$
$$\left(x + \sqrt[3]{2}\right)\left[x^2 - \sqrt[3]{2}x + \left(\sqrt[3]{2}\right)^2\right](x + 1)(x^2 - x + 1) = 0$$
$$x + \sqrt[3]{2} = 0 \implies x = -\sqrt[3]{2}$$
$$x^2 - \sqrt[3]{2}x + \left(\sqrt[3]{2}\right)^2 = 0 \implies \text{No real solution (by the Quadratic Formula)}$$
$$x + 1 = 0 \implies x = -1$$
$$x^2 - x + 1 = 0 \implies \text{No real solution (by the Quadratic Formula)}$$

153. $\sqrt{2x} - 10 = 0$

$\sqrt{2x} = 10$

$2x = 100$

$x = 50$

154. $4\sqrt{x} - 3 = 0$

$4\sqrt{x} = 3$

$16x = 9$

$x = \frac{9}{16}$

155. $\sqrt{x - 10} - 4 = 0$

$\sqrt{x - 10} = 4$

$x - 10 = 16$

$x = 26$

156. $\sqrt{5 - x} - 3 = 0$

$\sqrt{5 - x} = 3$

$5 - x = 9$

$x = -4$

157. $\sqrt[3]{2x + 5} + 3 = 0$

$\sqrt[3]{2x + 5} = -3$

$2x + 5 = -27$

$2x = -32$

$x = -16$

158. $\sqrt[3]{3x + 1} - 5 = 0$

$\sqrt[3]{3x + 1} = 5$

$3x + 1 = 125$

$3x = 124$

$x = \frac{124}{3}$

159. $-\sqrt{26 - 11x} + 4 = x$

$4 - x = \sqrt{26 - 11x}$

$16 - 8x + x^2 = 26 - 11x$

$x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$

$x + 5 = 0 \Rightarrow x = -5$

$x - 2 = 0 \Rightarrow x = 2$

160. $x + \sqrt{31 - 9x} = 5$

$\sqrt{31 - 9x} = 5 - x$

$31 - 9x = 25 - 10x + x^2$

$0 = x^2 - x - 6$

$0 = (x - 3)(x + 2)$

$0 = x - 3 \Rightarrow x = 3$

$0 = x + 2 \Rightarrow x = -2$

161. $\sqrt{x + 1} = \sqrt{3x + 1}$

$x + 1 = 3x + 1$

$-2x = 0$

$x = 0$

162. $\sqrt{x + 5} = \sqrt{x - 5}$

$x + 5 = x - 5$

$5 = -5$

No solution

163. $(x - 5)^{3/2} = 8$

$(x - 5)^3 = 8^2$

$x - 5 = \sqrt[3]{64}$

$x = 5 + 4 = 9$

164. $(x + 3)^{3/2} = 8$

$(x + 3)^3 = 8^2$

$(x + 3)^3 = 64$

$x + 3 = \sqrt[3]{64}$

$x = -3 + 4 = 1$

165. $(x + 3)^{2/3} = 8$

$(x + 3)^2 = 8^3$

$x + 3 = \pm\sqrt{8^3}$

$x + 3 = \pm\sqrt{512}$

$x = -3 \pm 16\sqrt{2}$

166. $(x + 2)^{2/3} = 9$

$(x + 2)^2 = 9^3$

$x + 2 = \pm\sqrt{729}$

$x = -2 \pm 27$

$= -29, 25$

167. $(x^2 - 5)^{3/2} = 27$

$(x^2 - 5)^3 = 27^2$

$x^2 - 5 = \sqrt[3]{27^2}$

$x^2 = 5 + 9$

$x^2 = 14$

$x = \pm\sqrt{14}$

168. $(x^2 - x - 22)^{3/2} = 27$

$$x^2 - x - 22 = 27^{2/3}$$
$$x^2 - x - 22 = 9$$
$$x^2 - x - 31 = 0$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-31)}}{2(1)} = \frac{1 \pm \sqrt{125}}{2} = \frac{1 \pm 5\sqrt{5}}{2}$$

169. $3x(x-1)^{1/2} + 2(x-1)^{3/2} = 0$

$$(x-1)^{1/2}[3x + 2(x-1)] = 0$$
$$(x-1)^{1/2}(5x - 2) = 0$$
$$(x-1)^{1/2} = 0 \implies x - 1 = 0 \implies x = 1$$
$$5x - 2 = 0 \implies x = \tfrac{2}{5}, \text{ extraneous}$$

170. $4x^2(x-1)^{1/3} + 6x(x-1)^{4/3} = 0$

$$2x[2x(x-1)^{1/3} + 3(x-1)^{4/3}] = 0$$
$$2x(x-1)^{1/3}[2x + 3(x-1)] = 0$$
$$2x(x-1)^{1/3}(5x - 3) = 0$$
$$2x = 0 \implies x = 0$$
$$x - 1 = 0 \implies x = 1$$
$$5x - 3 = 0 \implies x = \tfrac{3}{5}$$

171. $x = \frac{3}{x} + \frac{1}{2}$

$$(2x)(x) = (2x)\left(\frac{3}{x}\right) + (2x)\left(\frac{1}{2}\right)$$
$$2x^2 = 6 + x$$
$$2x^2 - x - 6 = 0$$
$$(2x + 3)(x - 2) = 0$$
$$2x + 3 = 0 \implies x = -\frac{3}{2}$$
$$x - 2 = 0 \implies x = 2$$

172. $\frac{4}{x} - \frac{5}{3} = \frac{x}{6}$

$$(6x)\frac{4}{x} - (6x)\frac{5}{3} = (6x)\frac{x}{6}$$
$$24 - 10x = x^2$$
$$x^2 + 10x - 24 = 0$$
$$(x + 12)(x - 2) = 0$$
$$x + 12 = 0 \implies x = -12$$
$$x - 2 = 0 \implies x = 2$$

173. $\frac{1}{x} - \frac{1}{x+1} = 3$

$$x(x+1)\frac{1}{x} - x(x+1)\frac{1}{x+1} = x(x+1)(3)$$
$$x + 1 - x = 3x(x + 1)$$
$$1 = 3x^2 + 3x$$
$$0 = 3x^2 + 3x - 1$$
$$a = 3, \quad b = 3, \quad c = -1$$
$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(3)(-1)}}{2(3)} = \frac{-3 \pm \sqrt{21}}{6}$$

174. $\frac{4}{x+1} - \frac{3}{x+2} = 1$

$$4(x + 2) - 3(x + 1) = (x + 1)(x + 2)$$
$$4x + 8 - 3x - 3 = x^2 + 3x + 2$$
$$x^2 + 2x - 3 = 0$$
$$(x - 1)(x + 3) = 0$$
$$x - 1 = 0 \implies x = 1$$
$$x + 3 = 0 \implies x = -3$$

175. $\dfrac{20-x}{x} = x$

$$20 - x = x^2$$
$$0 = x^2 + x - 20$$
$$0 = (x+5)(x-4)$$
$$x + 5 = 0 \Rightarrow x = -5$$
$$x - 4 = 0 \Rightarrow x = 4$$

176. $4x + 1 = \dfrac{3}{x}$

$$(x)4x + (x)1 = (x)\frac{3}{x}$$
$$4x^2 + x = 3$$
$$4x^2 + x - 3 = 0$$
$$(4x-3)(x+1) = 0$$
$$4x - 3 = 0 \Rightarrow x = \frac{3}{4}$$
$$x + 1 = 0 \Rightarrow x = -1$$

177. $\dfrac{x}{x^2-4} + \dfrac{1}{x+2} = 3$

$$(x+2)(x-2)\frac{x}{x^2-4} + (x+2)(x-2)\frac{1}{x+2} = 3(x+2)(x-2)$$
$$x + x - 2 = 3x^2 - 12$$
$$3x^2 - 2x - 10 = 0$$

$a = 3,\ b = -2,\ c = -10$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-10)}}{2(3)}$$
$$= \frac{2 \pm \sqrt{124}}{6} = \frac{2 \pm 2\sqrt{31}}{6} = \frac{1 \pm \sqrt{31}}{3}$$

178. $\dfrac{x+1}{3} - \dfrac{x+1}{x+2} = 0$

$$3(x+2)\frac{x+1}{3} - 3(x+2)\frac{x+1}{x+2} = 0$$
$$(x+2)(x+1) - 3(x+1) = 0$$
$$x^2 + 3x + 2 - 3x - 3 = 0$$
$$x^2 - 1 = 0$$
$$(x+1)(x-1) = 0$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x - 1 = 0 \Rightarrow x = 1$$

179. $|2x - 1| = 5$

$$2x - 1 = 5 \Rightarrow x = 3$$
$$-(2x-1) = 5 \Rightarrow x = -2$$

180. $|3x + 2| = 7$

$$3x + 2 = 7 \Rightarrow x = \tfrac{5}{3}$$
$$-(3x+2) = 7$$
$$-3x - 2 = 7 \Rightarrow x = -3$$

181. $|x| = x^2 + x - 3$

First equation:

$$x = x^2 + x - 3$$
$$x^2 - 3 = 0$$
$$x = \pm\sqrt{3}$$

Second equation:

$$-x = x^2 + x - 3$$
$$x^2 + 2x - 3 = 0$$
$$(x-1)(x+3) = 0$$
$$x - 1 = 0 \Rightarrow x = 1$$
$$x + 3 = 0 \Rightarrow x = -3$$

Only $x = \sqrt{3}$ and $x = -3$ are solutions to the original equation. $x = -\sqrt{3}$ and $x = 1$ are extraneous.

182. $|x^2 + 6x| = 3x + 18$

First equation:

$$x^2 + 6x = 3x + 18$$
$$x^2 + 3x - 18 = 0$$
$$(x - 3)(x + 6) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x + 6 = 0 \Rightarrow x = -6$$

Second equation:

$$-(x^2 + 6x) = 3x + 18$$
$$0 = x^2 + 9x + 18$$
$$0 = (x + 3)(x + 6)$$
$$0 = x + 3 \Rightarrow x = -3$$
$$0 = x + 6 \Rightarrow x = -6$$

The solutions to the original equation are $x = \pm 3$ and $x = -6$.

183. $|x + 1| = x^2 - 5$

First equation:

$$x + 1 = x^2 - 5$$
$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x + 2 = 0 \Rightarrow x = -2$$

Second equation:

$$-(x + 1) = x^2 - 5$$
$$-x - 1 = x^2 - 5$$
$$x^2 + x - 4 = 0$$
$$x = \frac{-1 \pm \sqrt{17}}{2}$$

Only $x = 3$ and $x = \dfrac{-1 - \sqrt{17}}{2}$ are solutions to the original equation. $x = -2$ and $x = \dfrac{-1 + \sqrt{17}}{2}$ are extraneous.

184. $|x - 10| = x^2 - 10x$

First equation:

$$x - 10 = x^2 - 10x$$
$$0 = x^2 - 11x + 10$$
$$0 = (x - 1)(x - 10)$$
$$0 = x - 1 \Rightarrow x = 1$$
$$0 = x - 10 \Rightarrow x = 10$$

Second equation:

$$-(x - 10) = x^2 - 10x$$
$$0 = x^2 - 9x - 10$$
$$0 = (x - 10)(x + 1)$$
$$0 = x - 10 \Rightarrow x = 10$$
$$0 = x + 1 \Rightarrow x = -1$$

The solutions to the original equation are $x = 10$ and $x = -1$. $x = 1$ is extraneous.

185. (a) Female: $y = 0.432x - 10.44$

For $y = 16$: $16 = 0.432x - 10.44$

$$26.44 = 0.432x$$
$$\frac{26.44}{0.432} = x$$
$$x \approx 61.2 \text{ inches}$$

(b) Male: $y = 0.449x - 12.15$

For $y = 19$: $19 = 0.449x - 12.15$

$$31.15 = 0.449x$$
$$69.4 \approx x$$

Yes, it is likely that both bones came from the same person because the estimated height of a male with a 19-inch thigh bone is 69.4 inches.

—CONTINUED—

185. —CONTINUED—

(c)

Height x	Female Femur Length	Male Femur Length
60	15.48	14.79
70	19.80	19.28
80	24.12	23.77
90	28.44	28.26
100	32.76	32.75
110	37.08	37.24

The lengths of the male and female femurs are approximately equal when the lengths are 100 inches.

186. $10{,}000 = 0.32m + 2500$

$7500 = 0.32m$

$\dfrac{7500}{0.32} = m$

$m = 23{,}437.5$ miles

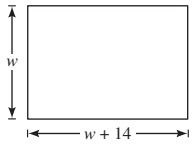
187. $y = -0.25t + 8$

$1 = -0.25t + 8$

$0.25t = 7$

$t = 28$ hours

188. (a)



(b) $w(w + 14) = 1632$

(c) $w^2 + 14w - 1632 = 0$

$(w + 48)(w - 34) = 0$

$w = -48$ or $w = 34$

Since w must be greater than zero, we have $w = 34$ feet and the length is $w + 14 = 48$ feet.

189. $S = x^2 + 4xh$

$84 = x^2 + 4x(2)$

$0 = x^2 + 8x - 84$

$0 = (x + 14)(x - 6)$

$x = -14$ or $x = 6$

Since x must be positive, we have $x = 6$ inches. The dimensions of the box are 6 inches $\times$ 6 inches $\times$ 2 inches.

190. $x^2 + x^2 = 5^2$ Pythagorean Theorem

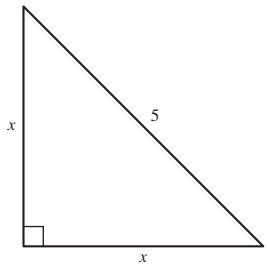
$2x^2 = 25$

$x^2 = \dfrac{25}{2}$

$x = \sqrt{\dfrac{25}{2}}$

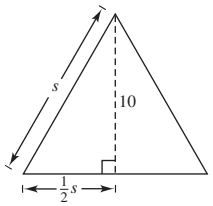
$= \dfrac{5}{\sqrt{2}}$

$= \dfrac{5\sqrt{2}}{2} \approx 3.54$ centimeters



Each leg in the right triangle is approximately 3.54 centimeters.

191. *Model*: $(\text{height})^2 + (\text{half of side})^2 = (\text{side})^2$



Labels: height $= 10$ inches, side $= s$, half of side $= \dfrac{s}{2}$

Equation: $10^2 + \left(\dfrac{s}{2}\right)^2 = s^2$

$$100 + \frac{s^2}{4} = s^2$$

$$\frac{3}{4}s^2 = 100$$

$$s^2 = \frac{400}{3}$$

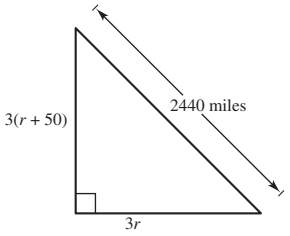
$$s = \sqrt{\frac{400}{3}} = \frac{20\sqrt{3}}{3} \approx 11.55 \text{ inches}$$

Each side of the equilateral triangle is approximately 11.55 inches long.

192.

$$d_N = (3 \text{ hours})(r + 50 \text{ mph})$$

$$d_E = (3 \text{ hours})(r \text{ mph})$$



$$d_N^2 + d_E^2 = 2440^2$$

$$9(r + 50)^2 + 9r^2 = 2440^2$$

$$18r^2 + 900r - 5{,}931{,}100 = 0$$

$$r = \frac{-900 \pm \sqrt{900^2 - 4(18)(-5{,}931{,}100)}}{2(18)} = \frac{-900 \pm 60\sqrt{118{,}847}}{36}$$

Using the positive value for r, we have one plane moving northbound at $r + 50 \approx 600$ miles per hour and one plane moving eastbound at $r \approx 550$ miles per hour.

193. (a) $P = 200$ million when:

$$\frac{182.45 - 3.189t}{1.00 - 0.026t} = 200$$

$$182.45 - 3.189t = 200(1.00 - 0.026t)$$

$$182.45 - 3.189t = 200 - 5.2t$$

$$2.011t = 17.55$$

$$t = 8.7$$

So the total voting-age population reached 200 million during 1998.

(b) For $P = 230$:

$$\frac{182.45 - 3.189t}{1.00 - 0.026t} = 230$$

$$182.45 - 3.189t = 230(1.00 - 0.026t)$$

$$182.45 - 3.189t = 230 - 5.98t$$

$$\text{so} \quad 2.791t = 47.55$$

$$t = 17$$

The model predicts that the total voting-age population will reach 230 million during 2007. This value is reasonable but the model is reaching its limit since it soon begins to rise very fast due to its asymptotic behavior.

194. When $C = 2.5$ we have:

$$2.5 = \sqrt{0.2x + 1}$$

$$6.25 = 0.2x + 1$$

$$5.25 = 0.2x$$

$$x = 26.25 = 26{,}250 \text{ passengers}$$

195.

$$37.55 = 40 - \sqrt{0.01x + 1}$$

$$\sqrt{0.01x + 1} = 2.45$$

$$0.01x + 1 = 6.0025$$

$$0.01x = 5.0025$$

$$x = 500.25$$

Rounding x to the nearest whole unit yields $x \approx 500$ units.

196. When $p = \$750$, we have:

$$\begin{aligned} 750 &= 800 - \sqrt{0.01x + 1} \\ 50 &= \sqrt{0.01x + 1} \\ 2500 &= 0.01x + 1 \\ 0.01x &= 2499 \\ x &= 249{,}900 \end{aligned}$$

So the demand is 249,900 units when the price is \$750.

197. False. $x(3 - x) = 10 \implies 3x - x^2 = 10$

This is a quadratic equation. The equation cannot be written in the form $ax + b = 0$.

198. False. The product must equal zero for the Zero-Factor property to be used.

199. False—See Example 14 on page A55.

200. False. $|x| = 0$ has only one solution to check, 0.

201. Equivalent equations are derived from the substitution principle and simplification techniques. They have the same solution(s).

$2x + 3 = 8$ and $2x = 5$ are equivalent equations.

202. To transform an equation into an equivalent equation, you should first remove symbols of grouping, combine like terms, and reduce fractions. Then, as needed, you may add (or subtract) the same quantity to (from) both sides of the equation, multiply (divide) both sides of the equation by the same nonzero quantity, or interchange the two sides of the equation.

203. The student should have subtracted $15x$ from both sides so that the equation is equal to zero. By factoring out an x, there are two solutions, $x = 0$ and $x = 6$.

204. $3(x + 4)^2 + (x + 4) - 2 = 0$

(a) Let $u = x + 4$

$$\begin{aligned} 3u^2 + u - 2 &= 0 \\ (3u - 2)(u + 1) &= 0 \\ 3u - 2 = 0 \quad &\text{or} \quad u + 1 = 0 \\ u = \tfrac{2}{3} \quad &\qquad u = -1 \\ x + 4 = \tfrac{2}{3} \quad &\qquad x + 4 = -1 \\ x = -\tfrac{10}{3} \quad &\text{or} \quad x = -5 \end{aligned}$$

(b) $3(x^2 + 8x + 16) + (x + 4) - 2 = 0$

$$\begin{aligned} 3x^2 + 24x + 48 + x + 4 - 2 &= 0 \\ 3x^2 + 25x + 50 &= 0 \\ (3x + 10)(x + 5) &= 0 \\ 3x + 10 = 0 \quad &\text{or} \quad x + 5 = 0 \\ x = -\tfrac{10}{3} \quad &\qquad x = -5 \end{aligned}$$

(c) The method of part (a) reduces the number of algebraic steps.

205. -3 and 6

One possible equation is:

$$\begin{aligned} (x - (-3))(x - 6) &= 0 \\ (x + 3)(x - 6) &= 0 \\ x^2 - 3x - 18 &= 0 \end{aligned}$$

Any non-zero multiple of this equation would also have these solutions.

206. $(x - (-4))(x - (-11)) = 0$

$$\begin{aligned} (x + 4)(x + 11) &= 0 \\ x^2 + 15x + 44 &= 0 \end{aligned}$$

207. 8 and 14

One possible equation is:

$(x - 8)(x - 14) = 0$

$x^2 - 22x + 112 = 0$

Any non-zero multiple of this equation would also have these solutions.

208. $x = \frac{1}{6} \Rightarrow 6x = 1 \Rightarrow 6x - 1$ is a factor.

$x = -\frac{2}{5} \Rightarrow 5x = -2 \Rightarrow 5x + 2$ is a factor.

$(6x - 1)(5x + 2) = 0$

$30x^2 + 7x - 2 = 0$

209. $1 + \sqrt{2}$ and $1 - \sqrt{2}$

One possible equation is:

$$\left[x - \left(1 + \sqrt{2}\right)\right]\left[x - \left(1 - \sqrt{2}\right)\right] = 0$$
$$\left[(x - 1) - \sqrt{2}\right]\left[(x - 1) + \sqrt{2}\right] = 0$$
$$(x - 1)^2 - \left(\sqrt{2}\right)^2 = 0$$
$$x^2 - 2x + 1 - 2 = 0$$
$$x^2 - 2x - 1 = 0$$

Any non-zero multiple of this equation would also have these solutions.

210. $x = -3 + \sqrt{5}$, $x = -3 - \sqrt{5}$, so:

$$\left(x - \left(-3 + \sqrt{5}\right)\right)\left(x - \left(-3 - \sqrt{5}\right)\right) = 0$$
$$\left(x + 3 - \sqrt{5}\right)\left(x + 3 + \sqrt{5}\right) = 0$$
$$x^2 + 6x + 4 = 0$$

211. $9 + |9 - a| = b$

$|9 - a| = b - 9$

$9 - a = b - 9$ OR $9 - a = -(b - 9)$

$-a = b - 18$ $\qquad 9 - a = -b + 9$

$a = 18 - b$ $\qquad -a = -b$

$a = b$

Thus, $a = 18 - b$ or $a = b$. From the original equation we know that $b \geq 9$.

Some possibilities are: $b = 9$, $a = 9$

$b = 10$, $a = 8$ or $a = 10$

$b = 11$, $a = 7$ or $a = 11$

$b = 12$, $a = 6$ or $a = 12$

$b = 13$, $a = 5$ or $a = 13$

$b = 14$, $a = 4$ or $a = 14$

212. Isolate the absolute value by subtracting x from both sides of the equation. The expression inside the absolute value signs can be positive or negative, so two separate equations must be solved. Each solution must be checked since extraneous solutions may be included.

213. (a) $ax^2 + bx = 0$

$x(ax + b) = 0$

$x = 0$

$ax + b = 0 \Rightarrow x = -\dfrac{b}{a}$

(b) $ax^2 - ax = 0$

$ax(x - 1) = 0$

$ax = 0 \Rightarrow x = 0$

$x - 1 = 0 \Rightarrow x = 1$

Appendix A.6 Linear Inequalities in One Variable

■ You should know the properties of inequalities.

(a) Transitive: $a < b$ and $b < c$ implies $a < c$.

(b) Addition: $a < b$ and $c < d$ implies $a + c < b + d$.

(c) Adding or Subtracting a Constant: $a \pm c < b \pm c$ if $a < b$.

(d) Multiplying or Dividing a Constant: For $a < b$,

1. If $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

2. If $c < 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

■ You should be able to solve absolute value inequalities.

(a) $|x| < a$ if and only if $-a < x < a$.

(b) $|x| > a$ if and only if $x < -a$ or $x > a$.

Vocabulary Check

1. solution set **2.** graph **3.** negative

4. equivalent **5.** double **6.** union

1. Interval: $[-1, 5]$

(a) Inequality: $-1 \le x \le 5$

(b) The interval is bounded.

2. Interval: $(2, 10]$

(a) Inequality: $2 < x \le 10$

(b) The interval is bounded.

3. Interval: $(11, \infty)$

(a) Inequality: $x > 11$

(b) The interval is unbounded.

4. Interval: $[-5, \infty)$

(a) Inequality: $-5 \le x < \infty$ or $x \ge -5$

(b) The interval is unbounded.

5. Interval: $(-\infty, -2)$

(a) Inequality: $x < -2$

(b) The interval is unbounded.

6. Interval: $(-\infty, 7]$

(a) Inequality: $-\infty < x \le 7$ or $x \le -7$

(b) The interval is unbounded.

7. $x < 3$

Matches (b).

8. $x \ge 5$

Matches (f).

9. $-3 < x \le 4$

Matches (d).

10. $0 \le x \le \frac{9}{2}$

Matches (c).

11. $|x| < 3 \implies -3 < x < 3$

Matches (e).

12. $|x| > 4 \implies x > 4$ or $x < -4$

Matches (a).

13. $5x - 12 > 0$

(a) $x = 3$

$5(3) - 12 \stackrel{?}{>} 0$

$3 > 0$

Yes, $x = 3$ *is* a solution.

(b) $x = -3$

$5(-3) - 12 \stackrel{?}{>} 0$

$-27 \not> 0$

No, $x = -3$ *is not* a solution.

(c) $x = \frac{5}{2}$

$5\left(\frac{5}{2}\right) - 12 \stackrel{?}{>} 0$

$\frac{1}{2} > 0$

Yes, $x = \frac{5}{2}$ *is* a solution.

(d) $x = \frac{3}{2}$

$5\left(\frac{3}{2}\right) - 12 \stackrel{?}{>} 0$

$-\frac{9}{2} \not> 0$

No, $x = \frac{3}{2}$ *is not* a solution.

14. $2x + 1 < -3$

(a) $x = 0$

$2(0) + 1 \stackrel{?}{<} -3$

$1 \not< -3$

No, $x = 0$ *is not* a solution.

(b) $x = -\frac{1}{4}$

$2\left(-\frac{1}{4}\right) + 1 \stackrel{?}{<} -3$

$\frac{1}{2} \not< -3$

No, $x = -\frac{1}{4}$ *is not* a solution.

(c) $x = -4$

$2(-4) + 1 \stackrel{?}{<} -3$

$-7 < -3$

Yes, $x = -4$ *is* a solution.

(d) $x = -\frac{3}{2}$

$2\left(-\frac{3}{2}\right) + 1 \stackrel{?}{<} -3$

$-2 \not< -3$

No, $x = -\frac{3}{2}$ *is not* a solution.

15. $0 < \dfrac{x - 2}{4} < 2$

(a) $x = 4$

$0 \stackrel{?}{<} \dfrac{4 - 2}{4} \stackrel{?}{<} 2$

$0 < \dfrac{1}{2} < 2$

Yes, $x = 4$ *is* a solution.

(b) $x = 10$

$0 \stackrel{?}{<} \dfrac{10 - 2}{4} \stackrel{?}{<} 2$

$0 < 2 \not< 2$

No, $x = 10$ *is not* a solution.

(c) $x = 0$

$0 \stackrel{?}{<} \dfrac{0 - 2}{4} \stackrel{?}{<} 2$

$0 \not< -\dfrac{1}{2} < 2$

No, $x = 0$ *is not* a solution.

(d) $x = \dfrac{7}{2}$

$0 \stackrel{?}{<} \dfrac{(7/2) - 2}{4} \stackrel{?}{<} 2$

$0 < \dfrac{3}{8} < 2$

Yes, $x = \frac{7}{2}$ *is* a solution.

16. $-1 < \dfrac{3 - x}{2} \leq 1$

(a) $x = 0$

$-1 \stackrel{?}{<} \dfrac{3 - 0}{2} \stackrel{?}{\leq} 1$

$-1 < \dfrac{3}{2} \not\leq 1$

No, $x = 0$ *is not* a solution.

(b) $x = -5$

$-1 \stackrel{?}{<} \dfrac{3 + 5}{2} \stackrel{?}{\leq} 1$

$-1 < 4 \leq 1$

No, $x = -5$ *is not* a solution.

(c) $x = 1$

$-1 \stackrel{?}{<} \dfrac{3 - 1}{2} \stackrel{?}{\leq} 1$

$-1 < 1 \leq 1$

Yes, $x = 1$ *is* a solution.

(d) $x = 5$

$-1 \stackrel{?}{<} \dfrac{3 - 5}{2} \stackrel{?}{\leq} 1$

$-1 \stackrel{?}{<} -1 \leq 1$

No, $x = 5$ *is not* a solution.

17. $|x - 10| \geq 3$

(a) $x = 13$

$|13 - 10| \stackrel{?}{\geq} 3$

$3 \geq 3$

Yes, $x = 13$ *is* a solution.

(b) $x = -1$

$|-1 - 10| \stackrel{?}{\geq} 3$

$11 \geq 3$

Yes, $x = -1$ *is* a solution.

(c) $x = 14$

$|14 - 10| \stackrel{?}{\geq} 3$

$4 \geq 3$

Yes, $x = 14$ *is* a solution.

(d) $x = 9$

$|9 - 10| \stackrel{?}{\geq} 3$

$1 \not\geq 3$

No, $x = 9$ *is not* a solution.

18. $|2x - 3| < 15$

(a) $x = -6$

$|2(-6) - 3| \stackrel{?}{<} 15$

$15 \not< 15$

No, $x = -6$ *is not* a solution.

(b) $x = 0$

$|2(0) - 3| \stackrel{?}{<} 15$

$3 < 15$

Yes, $x = 0$ *is* a solution.

(c) $x = 12$

$|2(12) - 3| \stackrel{?}{<} 15$

$21 \not< 15$

No, $x = 12$ *is not* a solution.

(d) $x = 7$

$|2(7) - 3| \stackrel{?}{<} 15$

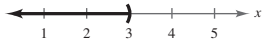
$11 < 15$

Yes, $x = 7$ *is* a solution.

19. $4x < 12$

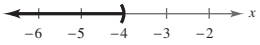
$\frac{1}{4}(4x) < \frac{1}{4}(12)$

$x < 3$



20. $10x < -40$

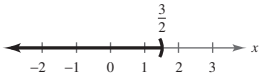
$x < -4$



21. $-2x > -3$

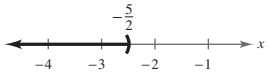
$-\frac{1}{2}(-2x) < \left(-\frac{1}{2}\right)(-3)$

$x < \frac{3}{2}$



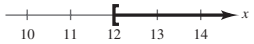
22. $-6x > 15$

$x < -\frac{15}{6}$ or $x < -\frac{5}{2}$



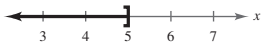
23. $x - 5 \geq 7$

$x \geq 12$



24. $x + 7 \leq 12$

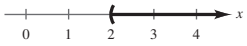
$x \leq 5$



25. $2x + 7 < 3 + 4x$

$-2x < -4$

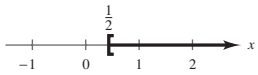
$x > 2$



26. $3x + 1 \geq 2 + x$

$2x \geq 1$

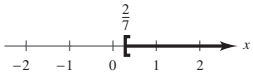
$x \geq \frac{1}{2}$



27. $2x - 1 \geq 1 - 5x$

$7x \geq 2$

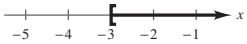
$x \geq \frac{2}{7}$



28. $6x - 4 \leq 2 + 8x$

$-2x \leq 6$

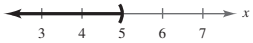
$x \geq -3$



29. $4 - 2x < 3(3 - x)$

$4 - 2x < 9 - 3x$

$x < 5$

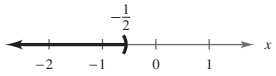


30. $4(x + 1) < 2x + 3$

$4x + 4 < 2x + 3$

$2x < -1$

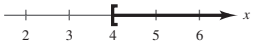
$x < -\frac{1}{2}$



31. $\frac{3}{4}x - 6 \leq x - 7$

$-\frac{1}{4}x \leq -1$

$x \geq 4$

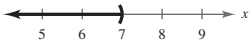


32. $3 + \frac{2}{7}x > x - 2$

$21 + 2x > 7x - 14$

$-5x > -35$

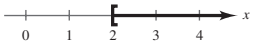
$x < 7$



33. $\frac{1}{2}(8x + 1) \geq 3x + \frac{5}{2}$

$4x + \frac{1}{2} \geq 3x + \frac{5}{2}$

$x \geq 2$

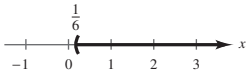


34. $9x - 1 < \frac{3}{4}(16x - 2)$

$36x - 4 < 48x - 6$

$-12x < -2$

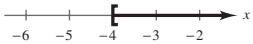
$x > \frac{1}{6}$



35. $3.6x + 11 \geq -3.4$

$3.6x \geq -14.4$

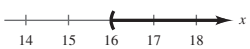
$x \geq -4$



36. $15.6 - 1.3x < -5.2$

$-1.3x < -20.8$

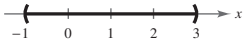
$x > 16$



37. $1 < 2x + 3 < 9$

$-2 < 2x < 6$

$-1 < x < 3$

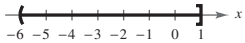


38. $-8 \le -(3x + 5) < 13$

$-8 \le -3x - 5 < 13$

$-3 \le -3x < 18$

$-6 < x \le 1$

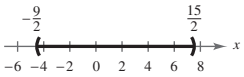


39. $-4 < \dfrac{2x - 3}{3} < 4$

$-12 < 2x - 3 < 12$

$-9 < 2x < 15$

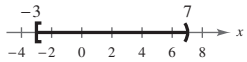
$-\frac{9}{2} < x < \frac{15}{2}$



40. $0 \le \dfrac{x + 3}{2} < 5$

$0 \le x + 3 < 10$

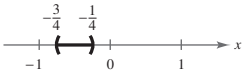
$-3 \le x < 7$



41. $\dfrac{3}{4} > x + 1 > \dfrac{1}{4}$

$-\frac{1}{4} > x > -\frac{3}{4}$

$-\frac{3}{4} < x < -\frac{1}{4}$

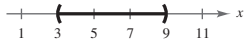


42. $-1 < 2 - \dfrac{x}{3} < 1$

$-3 < 6 - x < 3$

$-9 < -x < -3$

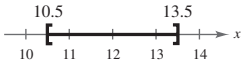
$3 < x < 9$



43. $3.2 \le 0.4x - 1 \le 4.4$

$4.2 \le 0.4x \le 5.4$

$10.5 \le x \le 13.5$



44. $4.5 > \dfrac{1.5x + 6}{2} > 10.5$

$9 > 1.5x + 6 > 21$

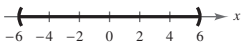
$3 > 1.5x > 15$

$2 > x > 10$

There is no solution.

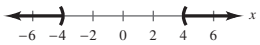
45. $|x| < 6$

$-6 < x < 6$



46. $|x| > 4$

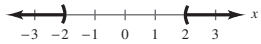
$x < -4$ or $x > 4$



47. $\left|\dfrac{x}{2}\right| > 1$

$\frac{x}{2} < -1$ or $\frac{x}{2} > 1$

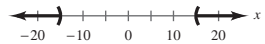
$x < -2 \quad x > 2$



48. $\left|\dfrac{x}{5}\right| > 3$

$\frac{x}{5} < -3$ or $\frac{x}{5} > 3$

$x < -15 \quad x > 15$



49. $|x - 5| < -1$

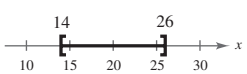
No solution. The absolute value of a number cannot be less than a negative number.

50. There is no solution because the absolute value of a number cannot be less than a negative number.

51. $|x - 20| \le 6$

$-6 \le x - 20 \le 6$

$14 \le x \le 26$



52. $|x - 8| \ge 0$

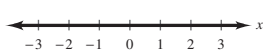
$x - 8 \ge 0$ or $-(x - 8) \ge 0$

$x \ge 8 \qquad -x + 8 \ge 0$

$-x \ge -8$

$x \le 8$

All real numbers x



53. $|3 - 4x| \geq 9$

$3 - 4x \leq -9$ or $3 - 4x \geq 9$

$-4x \leq -12$ $\quad -4x \geq 6$

$x \geq 3$ $\quad x \leq -\frac{3}{2}$

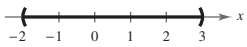
54. $|1 - 2x| < 5$

$-5 < 1 - 2x < 5$

$-6 < -2x < 4$

$3 > x > -2$

$-2 < x < 3$



55. $\left|\frac{x-3}{2}\right| \geq 4$

$\frac{x-3}{2} \leq -4$ or $\frac{x-3}{2} \geq 4$

$x - 3 \leq -8$ $\quad x - 3 \geq 8$

$x \leq -5$ $\quad x \geq 11$

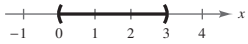
56. $\left|1 - \frac{2x}{3}\right| < 1$

$-1 < 1 - \frac{2x}{3} < 1$

$-2 < -\frac{2x}{3} < 0$

$3 > x > 0$

$0 < x < 3$



57. $|9 - 2x| - 2 < -1$

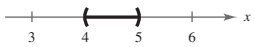
$|9 - 2x| < 1$

$-1 < 9 - 2x < 1$

$-10 < -2x < -8$

$5 > x > 4$

$4 < x < 5$

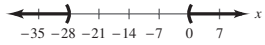


58. $|x + 14| + 3 > 17$

$|x + 14| > 14$

$x + 14 < -14$ or $x + 14 > 14$

$x < -28$ $\quad x > 0$

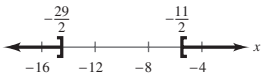


59. $2|x + 10| \geq 9$

$|x + 10| \geq \frac{9}{2}$

$x + 10 \leq -\frac{9}{2}$ or $x + 10 \geq \frac{9}{2}$

$x \leq -\frac{29}{2}$ $\quad x \geq -\frac{11}{2}$



60. $3|4 - 5x| \leq 9$

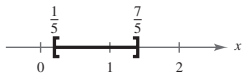
$|4 - 5x| \leq 3$

$-3 \leq 4 - 5x \leq 3$

$-7 \leq -5x \leq -1$

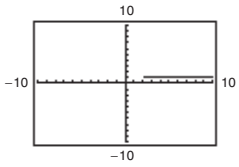
$\frac{7}{5} \geq x \geq \frac{1}{5}$

$\frac{1}{5} \leq x \leq \frac{7}{5}$



61. $6x > 12$

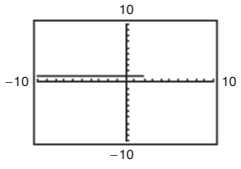
$x > 2$



62. $3x - 1 \leq 5$

$3x \leq 6$

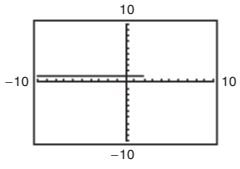
$x \leq 2$



63. $5 - 2x \geq 1$

$-2x \geq -4$

$x \leq 2$

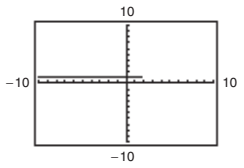


64. $3(x + 1) < x + 7$

$3x + 3 < x + 7$

$2x < 4$

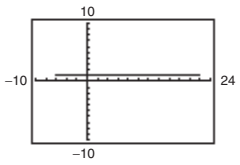
$x < 2$



65. $|x - 8| \le 14$

$-14 \le x - 8 \le 14$

$-6 \le x \le 22$

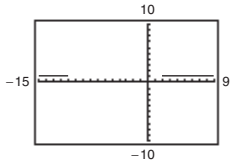


66. $|2x + 9| > 13$

$2x + 9 < -13$ or $2x + 9 > 13$

$2x < -22$ $\qquad$ $2x > 4$

$x < -11$ $\qquad$ $x > 2$

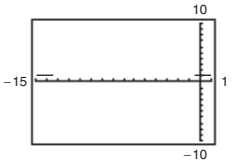


67. $2|x + 7| \ge 13$

$|x + 7| \ge \frac{13}{2}$

$x + 7 \le -\frac{13}{2}$ or $x + 7 \ge \frac{13}{2}$

$x \le -\frac{27}{2}$ $\qquad$ $x \ge -\frac{1}{2}$

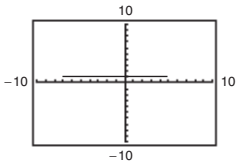


68. $\frac{1}{2}|x + 1| \le 3$

$|x + 1| \le 6$

$-6 \le x + 1 \le 6$

$-7 \le x \le 5$



69. $y = 2x - 3$

(a) $y \ge 1$

$2x - 3 \ge 1$

$2x \ge 4$

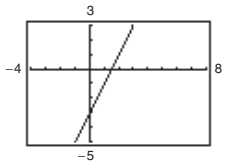
$x \ge 2$

(b) $y \le 0$

$2x - 3 \le 0$

$2x \le 3$

$x \le \frac{3}{2}$



70. $y = \frac{2}{3}x + 1$

(a) $y \le 5$

$\frac{2}{3}x + 1 \le 5$

$\frac{2}{3}x \le 4$

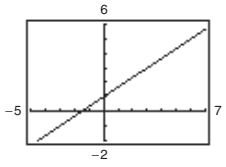
$x \le 6$

(b) $y \ge 0$

$\frac{2}{3}x + 1 \ge 0$

$\frac{2}{3}x \ge -1$

$x \ge -\frac{3}{2}$



71. $y = -\frac{1}{2}x + 2$

(a) $0 \le y \le 3$

$0 \le -\frac{1}{2}x + 2 \le 3$

$-2 \le -\frac{1}{2}x \le 1$

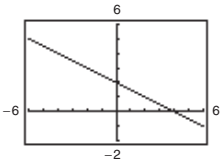
$4 \ge x \ge -2$

(b) $y \ge 0$

$-\frac{1}{2}x + 2 \ge 0$

$-\frac{1}{2}x \ge -2$

$x \le 4$



72. $y = -3x + 8$

(a) $-1 \le y \le 3$

$-1 \le -3x + 8 \le 3$

$-9 \le -3x \le -5$

$3 \ge x \ge \frac{5}{3}$

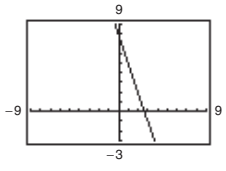
$\frac{5}{3} \le x \le 3$

(b) $y \le 0$

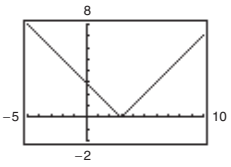
$-3x + 8 \le 0$

$-3x \le -8$

$x \ge \frac{8}{3}$



73. $y = |x - 3|$



(a) $y \le 2$

$|x - 3| \le 2$

$-2 \le x - 3 \le 2$

$1 \le x \le 5$

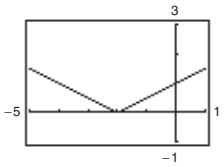
(b) $y \ge 4$

$|x - 3| \ge 4$

$x - 3 \le -4$ or $x - 3 \ge 4$

$x \le -1$ or $x \ge 7$

74. $y = \left|\frac{1}{2}x + 1\right|$



(a) $y \le 4$

$\left|\frac{1}{2}x + 1\right| \le 4$

$-4 \le \frac{1}{2}x + 1 \le 4$

$-5 \le \frac{1}{2}x \le 3$

$-10 \le x \le 6$

(b) $y \ge 1$

$\left|\frac{1}{2}x + 1\right| \ge 1$

$\frac{1}{2}x + 1 \le -1$ or $\frac{1}{2}x + 1 \ge 1$

$\frac{1}{2}x \le -2$ $\quad$ $\frac{1}{2}x \ge 0$

$x \le -4$ $\quad$ $x \ge 0$

75. $x - 5 \ge 0$

$x \ge 5$

$[5, \infty)$

76. $\sqrt{x - 10}$

$x - 10 \ge 0$

$x \ge 10$

$[10, \infty)$

77. $x + 3 \ge 0$

$x \ge -3$

$[-3, \infty)$

78. $\sqrt{3 - x}$

$3 - x \ge 0$

$3 \ge x$

$(-\infty, 3]$

79. $7 - 2x \ge 0$

$-2x \ge -7$

$x \le \frac{7}{2}$

$\left(-\infty, \frac{7}{2}\right]$

80. $\sqrt[4]{6x + 15}$

$6x + 15 \ge 0$

$6x \ge -15$

$x \ge -\frac{5}{2}$

$\left[-\frac{5}{2}, \infty\right)$

81. $|x - 10| < 8$

All real numbers within 8 units of 10.

82. $|x - 8| > 4$

All real numbers more than 4 units from 8

83. The midpoint of the interval $[-3, 3]$ is 0. The interval represents all real numbers x no more than 3 units from 0.

$|x - 0| \le 3$

$|x| \le 3$

84. The graph shows all real numbers more than 3 units from 0.

$|x - 0| > 3$

$|x| > 3$

85. The graph shows all real numbers at least 3 units from 7.

$|x - 7| \ge 3$

86. The graph shows all real numbers no more than 4 units from -1.

$|x + 1| \le 4$

87. All real numbers within 10 units of 12

$|x - 12| < 10$

88. All real numbers at least 5 units from 8

$|x - 8| \ge 5$

89. All real numbers more than 4 units from -3

$|x - (-3)| > 4$

$|x + 3| > 4$

90. All real numbers no more than 7 units from -6

$|x + 6| \le 7$

91. Let $x =$ the number of checks written in a month.

Type A account charges: $6.00 + 0.25x$

Type B account charges: $4.50 + 0.50x$

$$6.00 + 0.25x < 4.50 + 0.50x$$
$$1.50 < 0.25x$$
$$6 < x$$

If you write more than six checks a month, then the charges for the type A account are less than the charges for the type B account.

92. $3000 + 0.03x \le 0.1x$

$$3000 < 0.07x$$
$$42{,}857 < x$$

You must make more than 42,857 copies to justify buying the copier.

93. $1000(1 + r(2)) > 1062.50$

$$1 + 2r > 1.0625$$
$$2r > 0.0625$$
$$r > 0.03125$$
$$r > 3.125\%$$

94. $825 < 750(1 + r(2))$

$$825 < 750(1 + 2r)$$
$$825 < 750 + 1500r$$
$$75 < 1500r$$
$$0.05 < r$$

The rate must be more than 5%.

95. $R > C$

$$115.95x > 95x + 750$$
$$20.95x > 750$$
$$x > 35.7995$$
$$x \ge 36 \text{ units}$$

96. $24.55x > 15.4x + 150{,}000$

$$9.15 > 150{,}000$$
$$x > 16{,}393.44262$$

Because the number of units x must be an integer, the product will return a profit when at least 16,394 units are sold.

97. Let $x =$ daily sales level (in dozens) of doughnuts.

Revenue: $R = 2.95x$

Cost: $C = 150 + 1.45x$

Profit: $P = R - C$

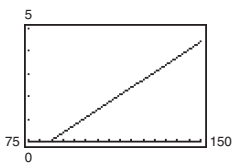
$$= 2.95x - (150 + 1.45x)$$
$$= 1.50x - 150$$
$$50 \le P \le 200$$
$$50 \le 1.50x - 150 \le 200$$
$$200 \le 1.50x \le 350$$
$$133\tfrac{1}{3} \le x \le 233\tfrac{1}{3}$$

In whole dozens, $134 \le x \le 234$.

98. The goal is to lose $164 - 128 = 36$ pounds. At $1\frac{1}{2}$ pounds per week, it will take 24 weeks.

$$36 \div 1\tfrac{1}{2} = 36 \times \tfrac{2}{3}$$
$$= 12 \times 2$$
$$= 24$$

99. (a) $y = 0.067x - 5.638$



(b) From the graph we see that $y \ge 3$ when $x \ge 129$. Algebraically we have:

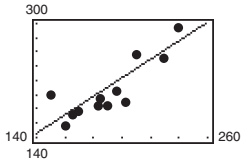
$$3 \le 0.067x - 5.638$$
$$8.638 \le 0.067x$$
$$x \ge 129$$

IQ scores are not a good predictor of GPAs. Other factors include study habits, class attendance, and attitude.

100. (a) and (b)

x	165	184	150	210	196	240
y	170	185	200	255	205	295
$1.3x - 36$	179	203	159	237	219	276

x	202	170	185	190	230	160
y	190	175	195	185	250	155
$1.3x - 36$	227	185	205	211	263	172



(c) One estimate is $x \geq 181$ pounds.

(d) $1.3x - 36 \geq 200$

$1.3x \geq 236$

$x \geq 181.5385 \approx 181.54$ pounds

(e) An athlete's weight is not a particularly good indicator of the athlete's maximum bench press weight. Other factors, such as muscle tone and exercise habits, influence maximum bench press weight.

101. $S = 1.05t + 31.0,\ 0 \leq t \leq 12$

(a) $32 \leq 1.05t + 31 \leq 42$

$1 \leq 1.05t \leq 11$

$0.95 \leq t \leq 10.48$

Rounding to the nearest year, $1 \leq t \leq 10$. The average salary was at least \$32,000 but not more than \$42,000 between 1991 and 2000.

(b) $1.05t + 31 > 48$

$1.05t > 17$

$t > 16$

According to the model, the average salary will exceed \$48,000 in 2006.

102. (a) $70 \leq 1.64t + 67.2 \leq 80$

$2.8 \leq 1.64t \leq 12.8$

$1.7 \leq t \leq 7.8$

The number of eggs produced was between 70 and 80 billion from late 1991 until late 1997.

(b) $E = 1.64t + 67.2 \geq 95$ when $t \geq 16.95$.

So the annual egg production will exceed 95 billion in 2007.

103. $|s - 10.4| \leq \frac{1}{16}$

$-\frac{1}{16} \leq s - 10.4 \leq \frac{1}{16}$

$-0.0625 \leq s - 10.4 \leq 0.0625$

$10.3375 \leq s \leq 10.4625$

Since $A = s^2$, we have

$(10.3375)^2 \leq \text{area} \leq (10.4625)^2$

$106.864 \leq \text{area} \leq 109.464.$

104. $24.2 - 0.25 \leq s \leq 24.2 + 0.25$

$23.95 \leq s \leq 24.45$

The interval containing the possible side lengths s in centimeters of the square is $[23.95, 24.45]$, so the interval containing the possible areas in square centimeters is $[23.95^2, 24.45^2]$, or $[573.6025, 597.8025]$.

105. $|x - 15| \leq \frac{1}{10}$

$-\frac{1}{10} \leq x - 15 \leq \frac{1}{10}$

$14.9 \leq x \leq 15.1$ gallons

$\frac{1}{10}(\$1.89) \approx \$0.19$

You might have been undercharged or overcharged by \$0.19.

106. 1 oz $= \frac{1}{16}$ lb, so $\frac{1}{2}$ oz $= \frac{1}{32}$ lb

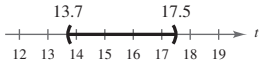
$14.99 \cdot \frac{1}{32} = 0.4684375$, so you could have been under- or overcharged as much as \$0.47.

107. $\left|\dfrac{t-15.6}{1.9}\right| < 1$

$$-1 < \frac{t-15.6}{1.9} < 1$$

$$-1.9 < t - 15.6 < 1.9$$

$$13.7 < t < 17.5$$



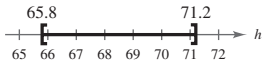
Two-thirds of the workers could perform the task in the time interval between 13.7 minutes and 17.5 minutes.

108. $\left|\dfrac{h-68.5}{2.7}\right| \le 1$

$$-1 \le \frac{h-68.5}{2.7} \le 1$$

$$-2.7 \le h - 68.5 \le 2.7$$

$$65.8 \text{ inches} \le h \le 71.2 \text{ inches}$$



109. $|h - 50| \le 30$

$$-30 \le h - 50 \le 30$$

$$20 \le h \le 80$$

The minimum relative humidity is 20 and the maximum is 80.

110. (a) Estimate from the graph: when the plate thickness is 2 millimeters, the frequency is approximately 330 vibrations per second.

(b) Estimate from the graph: when the frequency is 600, the plate thickness is approximately 3.6 millimeters.

(c) Estimate from the graph: when the frequency is between 200 and 400 vibrations per second, the plate thickness is between 1.2 and 2.4 millimeter.

(d) Estimate from the graph: when the plate thickness is less than 3 millimeters, the frequency is less than 500 vibrations per second.

111. False. If c is negative, then $ac \ge bc$.

112. False. If $-10 \le x \le 8$, then $10 \ge -x$ and $-x \ge -8$.

113. $|x - a| \ge 2$ Matches (b).

$$x - a \le -2$$

$$x \le a - 2 \text{ or}$$

$$x - a \ge 2$$

$$x \ge a + 2$$

114. $|ax - b| \le c \implies c$ must be greater than or equal to zero.

$$-c \le ax - b \le c$$

$$b - c \le ax \le b + c$$

Let $a = 1$, then $b - c = 0$ and $b + c = 10$. This is true when $b = c = 5$.
One set of values is $a = 1$, $b = 5$,$c = 5$.

(*Note:* This solution is not unique. Any positive multiple of these values will also work, such as $a = 2$, $b = c = 10$ or $a = 3$, $b = c = 15$.)

Appendix A.7 Errors and the Algebra of Calculus

- You should be able to recognize and avoid the common algebraic errors involving parentheses, fractions, exponents, radicals, and cancellation.
- You should be able to "unsimplify" algebraic expressions by the following methods.
  - (a) Unusual Factoring
  - (b) Rewriting with Negative Exponents
  - (c) Writing a Fraction as a Sum of Terms
  - (d) Inserting Factors or Terms

Vocabulary Check

1. numerator

2. reciprocal

1. $2x - (3y + 4) \neq 2x - 3y + 4$

Change all signs when distributing the minus sign.

$2x - (3y + 4) = 2x - 3y - 4$

2. $5z + 3(x - 2) \neq 5z + 3x - 2$

The 3 is distributed to both terms.

$5z + 3(x - 2) = 5z + 3x - 6$

3. $\dfrac{4}{16x - (2x + 1)} \neq \dfrac{4}{14x + 1}$

Change all signs when distributing the minus sign.

$\dfrac{4}{16x - (2x + 1)} = \dfrac{4}{16x - 2x - 1} = \dfrac{4}{14x - 1}$

4. $\dfrac{1 - x}{(5 - x)(-x)} \neq \dfrac{x - 1}{x(x - 5)}$

The expression on the right should be negative.

$\dfrac{1 - x}{(5 - x)(-x)} = -\dfrac{x - 1}{x(x - 5)}$

5. $(5z)(6z) \neq 30z$

z occurs twice as a factor.

$(5z)(6z) = 30z^2$

6. $x(yz) \neq (xy)(xz)$

yz is one term, not two.

$x(yz) = xyz$

7. $a\left(\dfrac{x}{y}\right) \neq \dfrac{ax}{ay}$

The fraction as a whole is multiplied by a, not the numerator and denominator separately.

$a\left(\dfrac{x}{y}\right) = \dfrac{a}{1} \cdot \dfrac{x}{y} = \dfrac{ax}{y}$

8. $(4x)^2 \neq 4x^2$

The exponent applies to the coefficient also.

$(4x)^2 = 16x^2$

9. $\sqrt{x + 9} \neq \sqrt{x} + 3$

Do not apply the radical to the terms.

$\sqrt{x + 9}$ does not simplify.

10. $\sqrt{25 - x^2} \neq 5 - x$

Do not apply radicals term-by-term.

$\sqrt{25 - x^2} = \sqrt{(5 + x)(5 - x)}$

11. $\dfrac{2x^2 + 1}{5x} \neq \dfrac{2x + 1}{5}$

Divide out common factors not common terms.

$\dfrac{2x^2 + 1}{5x}$ cannot be simplified.

12. $\dfrac{6x + y}{6x - y}$ does not simplify.

Reduce common factors of the numerator and denominator, not common factors of terms.

13. $\dfrac{1}{a^{-1}+b^{-1}} \neq \left(\dfrac{1}{a+b}\right)^{-1}$

To get rid of negative exponents:

$$\frac{1}{a^{-1}+b^{-1}} = \frac{1}{a^{-1}+b^{-1}} \cdot \frac{ab}{ab} = \frac{ab}{b+a}$$

14. $\dfrac{1}{x+y^{-1}} \neq \dfrac{y}{x+1}$

The negative exponent is on a term of the denominator, not a factor.

$$\frac{1}{x+y^{-1}} = \frac{1}{x+(1/y)} \cdot \frac{y}{y} = \frac{y}{xy+1}$$

15. $(x^2+5x)^{1/2} \neq x(x+5)^{1/2}$

Factor within grouping symbols before applying the exponent to each factor.

$$(x^2+5x)^{1/2} = [x(x+5)]^{1/2} = x^{1/2}(x+5)^{1/2}$$

16. $x(2x-1)^2 \neq (2x^2-x)^2$

Factor within grouping symbols before applying the exponent to each factor.

$$x(2x-1)^2 = x(4x^2-4x+1)$$

17. $\dfrac{3}{x}+\dfrac{4}{y} = \dfrac{3}{x}\cdot\dfrac{y}{y}+\dfrac{4}{y}\cdot\dfrac{x}{x} = \dfrac{3y+4x}{xy}$

To add fractions, they must have a common denominator.

18. $\dfrac{1}{2y} = \left(\dfrac{1}{2}\right)y$

Be careful when using a slash to denote division.

$$\left(\frac{1}{2}\right)y = \frac{1}{2}\cdot y = \frac{y}{2}$$

19. $\dfrac{3x+2}{5} = \dfrac{1}{5}(3x+2)$

The required factor is $3x+2$.

20. $\dfrac{7x^2}{10} = \dfrac{7}{10}(x^2)$

The required factor is x^2.

21. $\frac{2}{3}x^2+\frac{1}{3}x+5 = \frac{2}{3}x^2+\frac{1}{3}x+\frac{15}{3} = \frac{1}{3}(2x^2+x+15)$

The required factor is $2x^2+x+15$.

22. $\frac{3}{4}x+\frac{1}{2} = \frac{3}{4}x+\frac{2}{4} = \frac{1}{4}(3x+2)$

The required factor is $3x+2$.

23. $x^2(x^3-1)^4 = \dfrac{1}{3}(x^3-1)^4(3x^2)$

The required factor is $\frac{1}{3}$.

24. $x(1-2x^2)^3 = \dfrac{-4x}{-4}(1-2x^2)^3 = \left(-\dfrac{1}{4}\right)(-4x)(1-2x^2)^3$

$$= \left(-\frac{1}{4}\right)(1-2x^2)^3(-4x)$$

The required factor is $-\frac{1}{4}$.

25. $\dfrac{4x+6}{(x^2+3x+7)^3} = \dfrac{2(2x+3)}{(x^2+3x+7)^3} = \dfrac{2}{1}\cdot\dfrac{(2x+3)}{1}\cdot\dfrac{1}{(x^2+3x+7)^3} = (2)\dfrac{1}{(x^2+3x+7)^3}(2x+3)$

The required factor is 2.

26. $\dfrac{x+1}{(x^2+2x-3)^2} = \dfrac{1}{2}\cdot\dfrac{2(x+1)}{(x^2+2x-3)^2}$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{(x^2+2x-3)^2}\right)(2x+2)$$

The required factor is $\frac{1}{2}$.

27. $\dfrac{3}{x}+\dfrac{5}{2x^2}-\dfrac{3}{2}x = \dfrac{6x}{2x^2}+\dfrac{5}{2x^2}-\dfrac{3x^3}{2x^2}$

$$= \left(\frac{1}{2x^2}\right)(6x+5-3x^3)$$

The required factor is $\dfrac{1}{2x^2}$.

28. $\dfrac{(x-1)^2}{169}+(y+5)^2=\dfrac{(x-1)(x-1)^2}{(x-1)(169)}+(y+5)^2$

$=\dfrac{(x-1)^3}{169(x-1)}+(y+5)^2$

The required factor is $(x-1)$.

29. $\dfrac{9x^2}{25}+\dfrac{16y^2}{49}=\dfrac{9}{25}\cdot\dfrac{x^2}{1}+\dfrac{16}{49}\cdot\dfrac{y^2}{1}$

$=\dfrac{1}{25/9}\cdot\dfrac{x^2}{1}+\dfrac{1}{49/16}\cdot\dfrac{y^2}{1}$

$=\dfrac{x^2}{(25/9)}+\dfrac{y^2}{(49/16)}$

The required factors are $\frac{25}{9}$ and $\frac{49}{16}$.

30. $\dfrac{3x^2}{4}-\dfrac{9y^2}{16}=\dfrac{\left(\frac{1}{3}\right)3x^2}{\left(\frac{1}{3}\right)4}-\dfrac{\left(\frac{1}{9}\right)9y^2}{\left(\frac{1}{9}\right)16}=\dfrac{x^2}{\frac{4}{3}}-\dfrac{y^2}{\frac{16}{9}}$

The required factors are $\frac{4}{3}$ and $\frac{16}{9}$.

31. $\dfrac{x^2}{1/12}-\dfrac{y^2}{2/3}=x^2\left(\dfrac{12}{1}\right)-y^2\left(\dfrac{3}{2}\right)=\dfrac{12x^2}{1}-\dfrac{3y^2}{2}$

The required factors are 1 and 2.

32. $\dfrac{x^2}{4/9}+\dfrac{y^2}{7/8}=x^2\left(\dfrac{9}{4}\right)+y^2\left(\dfrac{8}{7}\right)=\dfrac{9x^2}{4}+\dfrac{8y^2}{7}$

The required factors are 4 and 7.

33. $x^{1/3}-5x^{4/3}=x^{1/3}(1-5x^{3/3})=x^{1/3}(1-5x)$

The required factor is $1-5x$.

34. $3(2x+1)x^{1/2}+4x^{3/2}=x^{1/2}[3(2x+1)+4x]$

$=x^{1/2}(6x+3+4x)$

$=x^{1/2}(10x+3)$

The required factor is $10x+3$.

35. $(1-3x)^{4/3}-4x(1-3x)^{1/3}=(1-3x)^{1/3}[(1-3x)^1-4x]$

$=(1-3x)^{1/3}(1-7x)$

The required factor is $1-7x$.

36. $\dfrac{1}{2\sqrt{x}}+5x^{3/2}-10x^{5/2}=\dfrac{1}{2\sqrt{x}}+\dfrac{5x^{3/2}\left(2\sqrt{x}\right)}{2\sqrt{x}}-\dfrac{10x^{5/2}\left(2\sqrt{x}\right)}{2\sqrt{x}}$

$=\dfrac{1}{2\sqrt{x}}\left(1+10x^{3/2}\sqrt{x}-20x^{5/2}\sqrt{x}\right)$

$=\dfrac{1}{2\sqrt{x}}(1+10x^2-20x^3)$

The required factor is $(1+10x^2-20x^3)$.

37. $\frac{1}{10}(2x+1)^{5/2}-\frac{1}{6}(2x+1)^{3/2}=\frac{3}{30}(2x+1)^{3/2}(2x+1)^1-\frac{5}{30}(2x+1)^{3/2}$

$=\frac{1}{30}(2x+1)^{3/2}[3(2x+1)-5]$

$=\frac{1}{30}(2x+1)^{3/2}(6x-2)$

$=\frac{1}{30}(2x+1)^{3/2}2(3x-1)$

$=\frac{1}{15}(2x+1)^{3/2}(3x-1)$

The required factor is $3x-1$.

38. $\dfrac{3}{7}(t+1)^{7/3}-\dfrac{3}{4}(t+1)^{4/3}=\dfrac{12}{28}(t+1)^{4/3}(t+1)^{3/3}-\dfrac{21}{28}(t+1)^{4/3}$

$=\dfrac{3(t+1)^{4/3}}{28}[4(t+1)-7]$

$=\dfrac{3(t+1)^{4/3}}{28}(4t-3)$

The required factor is $(4t-3)$.

39. $\dfrac{3x^2}{(2x-1)^3} = 3x^2(2x-1)^{-3}$

40. $\dfrac{x+1}{x(6-x)^{1/2}} = (x+1)(x^{-1})(6-x)^{-1/2}$

41. $\dfrac{4}{3x} + \dfrac{4}{x^4} - \dfrac{7x}{\sqrt[3]{2x}} = 4(3x)^{-1} + 4x^{-4} - 7x(2x)^{-1/3}$

42. $\dfrac{x}{x-2} + \dfrac{1}{x^2} + \dfrac{8}{3(9x)^3} = x(x-2)^{-1} + x^{-2} + \dfrac{8}{3}(9x)^{-3}$

43. $\dfrac{16-5x-x^2}{x} = \dfrac{16}{x} - \dfrac{5x}{x} - \dfrac{x^2}{x} = \dfrac{16}{x} - 5 - x$

44. $\dfrac{x^3-5x^2+4}{x^2} = \dfrac{x^3}{x^2} - \dfrac{5x^2}{x^2} + \dfrac{4}{x^2} = x - 5 + \dfrac{4}{x^2}$

45.
$$\begin{aligned}\frac{4x^3-7x^2+1}{x^{1/3}} &= \frac{4x^3}{x^{1/3}} - \frac{7x^2}{x^{1/3}} + \frac{1}{x^{1/3}} \\ &= 4x^{3-1/3} - 7x^{2-1/3} + \frac{1}{x^{1/3}} \\ &= 4x^{8/3} - 7x^{5/3} + \frac{1}{x^{1/3}}\end{aligned}$$

46.
$$\begin{aligned}\frac{2x^5-3x^3+5x-1}{x^{3/2}} &= \frac{2x^5}{x^{3/2}} - \frac{3x^3}{x^{3/2}} + \frac{5x}{x^{3/2}} - \frac{1}{x^{3/2}} \\ &= 2x^{5-3/2} - 3x^{3-3/2} + 5x^{1-3/2} - x^{-3/2} \\ &= 2x^{7/2} - 3x^{3/2} + \frac{5}{x^{1/2}} - \frac{1}{x^{3/2}}\end{aligned}$$

47.
$$\begin{aligned}\frac{3-5x^2-x^4}{\sqrt{x}} &= \frac{3}{\sqrt{x}} - \frac{5x^2}{\sqrt{x}} - \frac{x^4}{\sqrt{x}} \\ &= \frac{3}{\sqrt{x}} - 5x^{2-1/2} - x^{4-1/2} \\ &= \frac{3}{x^{1/2}} - 5x^{3/2} - x^{7/2}\end{aligned}$$

48. $\dfrac{x^3-5x^4}{3x^2} = \dfrac{x^3}{3x^2} - \dfrac{5x^4}{3x^2} = \dfrac{x}{3} - \dfrac{5x^2}{3}$

49.
$$\begin{aligned}\frac{-2(x^2-3)^{-3}(2x)(x+1)^3 - 3(x+1)^2(x^2-3)^{-2}}{[(x+1)^3]^2} &= \frac{(x^2-3)^{-3}(x+1)^2[-4x(x+1) - 3(x^2-3)]}{(x+1)^6} \\ &= \frac{-4x^2-4x-3x^2+9}{(x^2-3)^3(x+1)^4} \\ &= \frac{-7x^2-4x+9}{(x^2-3)^3(x+1)^4}\end{aligned}$$

50.
$$\begin{aligned}\frac{x^5(-3)(x^2+1)^{-4}(2x) - (x^2+1)^{-3}(5)x^4}{(x^5)^2} &= \frac{x^4(x^2+1)^{-4}[-6x^2 - 5(x^2+1)]}{x^{10}} \\ &= \frac{x^4(-6x^2-5x^2-5)}{x^{10}(x^2+1)^4} \\ &= \frac{-11x^2-5}{x^6(x^2+1)^4}\end{aligned}$$

51.
$$\begin{aligned}\frac{(6x+1)^3(27x^2+2) - (9x^3+2x)(3)(6x+1)^2(6)}{[(6x+1)^3]^2} &= \frac{(6x+1)^2[(6x+1)(27x^2+2) - 18(9x^3+2x)]}{(6x+1)^6} \\ &= \frac{162x^3+12x+27x^2+2-162x^3-36x}{(6x+1)^4} \\ &= \frac{27x^2-24x+2}{(6x+1)^4}\end{aligned}$$

52. $$\frac{(4x^2+9)^{1/2}(2)-(2x+3)\left(\frac{1}{2}\right)(4x^2+9)^{-1/2}(8x)}{[(4x^2+9)^{1/2}]^2}=\frac{2(4x^2+9)^{-1/2}[(4x^2+9)-2x(2x+3)]}{(4x^2+9)}$$
$$=\frac{2(4x^2+9-4x^2-6x)}{(4x^2+9)^{3/2}}$$
$$=\frac{2(9-6x)}{(4x^2+9)^{3/2}}$$
$$=\frac{-6(2x-3)}{(4x^2+9)^{3/2}}$$

53. $$\frac{(x+2)^{3/4}(x+3)^{-2/3}-(x+3)^{1/3}(x+2)^{-1/4}}{[(x+2)^{3/4}]^2}=\frac{(x+2)^{-1/4}(x+3)^{-2/3}[(x+2)-(x+3)]}{(x+2)^{6/4}}$$
$$=\frac{x+2-x-3}{(x+2)^{1/4}(x+3)^{2/3}(x+2)^{6/4}}$$
$$=-\frac{1}{(x+3)^{2/3}(x+2)^{7/4}}$$

54. $$(2x-1)^{1/2}-(x+2)(2x-1)^{-1/2}=(2x-1)^{1/2}-\frac{(x+2)}{(2x-1)^{1/2}}$$
$$=\frac{2x-1}{(2x-1)^{1/2}}-\frac{(x+2)}{(2x-1)^{1/2}}$$
$$=\frac{2x-1-x-2}{(2x-1)^{1/2}}$$
$$=\frac{x-3}{(2x-1)^{1/2}}$$

55. $$\frac{2(3x-1)^{1/3}-(2x+1)(1/3)(3x-1)^{-2/3}(3)}{(3x-1)^{2/3}}=\frac{(3x-1)^{-2/3}[2(3x-1)-(2x+1)]}{(3x-1)^{2/3}}$$
$$=\frac{6x-2-2x-1}{(3x-1)^{2/3}(3x-1)^{2/3}}$$
$$=\frac{4x-3}{(3x-1)^{4/3}}$$

56. $$\frac{(x+1)(1/2)(2x-3x^2)^{-1/2}(2-6x)-(2x-3x^2)^{1/2}}{(x+1)^2}=\frac{(x+1)(2x-3x^2)^{-1/2}(1-3x)-(2x-3x^2)^{1/2}}{(x+1)^2}$$
$$=\frac{(2x-3x^2)^{-1/2}[(x+1)(1-3x)-(2x-3x^2)]}{(x+1)^2}$$
$$=\frac{x-3x^2+1-3x-2x+3x^2}{(2x-3x^2)^{1/2}(x+1)^2}$$
$$=\frac{1-4x}{(2x-3x^2)^{1/2}\,(x+1)^2}$$

57. $\dfrac{1}{(x^2+4)^{1/2}} \cdot \dfrac{1}{2}(x^2+4)^{-1/2}(2x) = \dfrac{1}{(x^2+4)^{1/2}} \cdot \dfrac{1}{(x^2+4)^{1/2}} \cdot \dfrac{1}{2}(2x)$

$$= \frac{1}{(x^2+4)^1}(x)$$

$$= \frac{x}{x^2+4}$$

58. $\dfrac{1}{x^2-6}(2x) + \dfrac{1}{2x+5}(2) = \dfrac{2x(2x+5)+2(x^2-6)}{(x^2-6)(2x+5)}$

$$= \frac{4x^2+10x+2x^2-12}{(x^2-6)(2x+5)} = \frac{6x^2+10x-12}{(x^2-6)(2x+5)}$$

$$= \frac{2(3x^2+5x-6)}{(x^2-6)(2x+5)}$$

59. $(x^2+5)^{1/2}\left(\dfrac{3}{2}\right)(3x-2)^{1/2}(3) + (3x-2)^{3/2}\left(\dfrac{1}{2}\right)(x^2+5)^{-1/2}(2x) = \dfrac{9}{2}(x^2+5)^{1/2}(3x-2)^{1/2} + x(x^2+5)^{-1/2}(3x-2)^{3/2}$

$$= \frac{9}{2}(x^2+5)^{1/2}(3x-2)^{1/2} + \frac{2}{2}x(x^2+5)^{-1/2}(3x-2)^{3/2}$$

$$= \frac{1}{2}(x^2+5)^{-1/2}(3x-2)^{1/2}[9(x^2+5)^1 + 2x(3x-2)^1]$$

$$= \frac{1}{2}(x^2+5)^{-1/2}(3x-2)^{1/2}(9x^2+45+6x^2-4x)$$

$$= \frac{(3x-2)^{1/2}(15x^2-4x+45)}{2(x^2+5)^{1/2}}$$

60. $(3x+2)^{-1/2}(3)(x-6)^{1/2}(1) + (x-6)^3\left(-\dfrac{1}{2}\right)(3x+2)^{-3/2}(3) = 3(3x+2)^{-1/2}(x-6)^{1/2} + \left(\dfrac{-3}{2}\right)(x-6)^3(3x+2)^{-3/2}$

$$= \frac{3}{2}(x-6)^{1/2}(3x+2)^{-3/2}[2(3x+2) - (x-6)^{5/2}]$$

$$= \frac{3(x-6)^{1/2}[6x+4-(x-6)^{5/2}]}{2(3x+2)^{3/2}}$$

61. $t = \dfrac{\sqrt{x^2+4}}{2} + \dfrac{\sqrt{(4-x)^2+4}}{6}$

(a)

x	t
0.5	1.70
1.0	1.72
1.5	1.78
2.0	1.89
2.5	2.02
3.0	2.18
3.5	2.36
4.0	2.57

(b) She should swim to a point about $\frac{1}{2}$ mile down the coast to minimize the time required to reach the finish line.

(c) $\dfrac{1}{2}x(x^2+4)^{-1/2} + \dfrac{1}{6}(x-4)(x^2-8x+20)^{-1/2} = \dfrac{3}{6}x(x^2+4)^{-1/2} + \dfrac{1}{6}(x-4)(x^2-8x+20)^{-1/2}$

$$= \frac{1}{6}[3x(x^2+4)^{-1/2} + (x-4)(x^2-8x+20)^{-1/2}$$

$$= \frac{1}{6}\left[\frac{3x}{(x^2+4)^{1/2}} + \frac{x-4}{(x^2-8x+20)^{1/2}}\right]$$

$$= \frac{3x\sqrt{x^2-8x+20} + (x-4)\sqrt{x^2+4}}{6\sqrt{x^2+4}\,\sqrt{x^2-8x+20}}$$

62. (a) $y_1 = x^2\left(\frac{1}{3}\right)(x^2 + 1)^{-2/3}(2x) + (x^2 + 1)^{1/3}(2x)$

$$= 2x(x^2 + 1)^{-2/3}\left[\frac{x^2}{3} + (x^2 + 1)\right]$$

$$= 2x(x^2 + 1)^{-2/3}\left[\frac{x^2}{3} + \frac{3(x^2 + 1)}{3}\right]$$

$$= \frac{2x}{(x^2 + 1)^{2/3}} \cdot \frac{4x^2 + 3}{3}$$

$$= \frac{2x(4x^2 + 3)}{3(x^2 + 1)^{2/3}}$$

$$= y_2$$

(b)

x	-2	-1	$-\frac{1}{2}$	0	1	2	$\frac{5}{2}$
y_1	-8.7	-2.9	-1.1	0	2.9	8.7	12.5
y_2	-8.7	-2.9	-1.1	0	2.9	8.7	12.5

63. True.

$$x^{-1} + y^{-2} = \frac{1}{x} + \frac{1}{y^2} = \frac{y^2 + x}{xy^2}$$

64. False. Cannot move term-by-term from denominator to numerator.

$$\frac{1}{x^{-2} + y^{-1}} = \frac{1}{\frac{1}{x^2} + \frac{1}{y}} = \frac{1}{\frac{y + x^2}{x^2y}} = \frac{x^2y}{y + x^2}$$

65. True.

$$\frac{1}{\sqrt{x} + 4} = \frac{1}{\sqrt{x} + 4} \cdot \frac{\sqrt{x} - 4}{\sqrt{x} - 4} = \frac{\sqrt{x} - 4}{x - 16}$$

66. False. $x^2 - 9$ does not factor into $(\sqrt{x} + 3)(\sqrt{x} - 3)$.

$$\frac{x^2 - 9}{\sqrt{x} - 3} = \frac{(x + 3)(x - 3)}{\sqrt{x} - 3} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$$

$$= \frac{(x + 3)(x - 3)(\sqrt{x} + 3)}{x - 9}$$

67. $x^n \cdot x^{3n} \neq x^{3n^2}$

Add exponents when multiplying powers with like bases.

$x^n \cdot x^{3n} = x^{4n}$

68. $(x^n)^{2n} + (x^{2n})^n = x^{2n^2} + x^{2n^2} = 2x^{2n^2}$

There is no error.

69. $x^{2n} + y^{2n} \neq (x^n + y^n)^2$

When squaring binomials, there is also a middle term.

$(x^n + y^n)^2 = x^{2n} + 2x^ny^n + y^{2n}$

70. $\frac{x^{2n} \cdot x^{3n}}{x^{3n} + x^2} = \frac{x^{2n+3n}}{x^{3n} + x^2} = \frac{x^{5n}}{x^{3n} + x^2}$

There is no error.

71. The two answers are equivalent and can be obtained by factoring.

$$\tfrac{1}{10}(2x - 1)^{5/2} + \tfrac{1}{6}(2x - 1)^{3/2} = \tfrac{1}{60}(2x - 1)^{3/2}[6(2x - 1) + 10]$$

$$= \tfrac{1}{60}(2x - 1)^{3/2}(12x + 4)$$

$$= \tfrac{4}{60}(2x - 1)^{3/2}(3x + 1)$$

$$= \tfrac{1}{15}(2x - 1)^{3/2}(3x + 1)$$

(a) $\tfrac{2}{3}x(2x - 3)^{3/2} - \tfrac{2}{15}(2x - 3)^{5/2} = \tfrac{2}{15}(2x - 3)^{3/2}[5x - (2x - 3)]$

$$= \tfrac{2}{15}(2x - 3)^{3/2}(3x + 3)$$

$$= \tfrac{2}{15}(2x - 3)^{3/2}3(x + 1)$$

$$= \tfrac{2}{5}(2x - 3)^{3/2}(x + 1)$$

(b) $\tfrac{2}{3}x(4 + x)^{3/2} - \tfrac{2}{15}(4 + x)^{5/2} = \tfrac{2}{15}(4 + x)^{3/2}[5x - (4 + x)]$

$$= \tfrac{2}{15}(4 + x)^{3/2}(4x - 4) = \tfrac{2}{15}(4 + x)^{3/2}4(x - 1) = \tfrac{8}{15}(4 + x)^{3/2}(x - 1)$$